CHAPTER 11

CHAPTER 11 Surface Area and Volume

Chapter Outline

In this chapter we extend what we know about two-dimensional figures to three-dimensional shapes. First, we will determine the parts and different types of 3D shapes. Then, we will find the surface area and volume of prisms, cylinders, pyramids, cones, and spheres. Lastly, we will expand what we know about similar shapes and their areas to similar solids and their volumes.

11.1 Exploring Solids

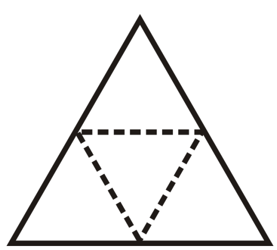
Learning Objectives

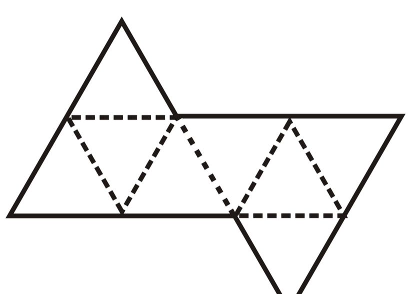
- Identify different types of solids and their parts.
- Use Euler's formula to solve problems.
- Draw and identify different views of solids.
- Draw and identify nets.

Review Queue

a. Draw an octagon and identify the edges and vertices of the octagon. How many of each are there?
b. Find the area of a square with 5 cm sides.
c. Find the area of an equilateral triangle with 10 in sides.
d. Draw the following polygons.
    a. A convex pentagon.
    b. A concave nonagon.

Know What? Until now, we have only talked about two-dimensional, or flat, shapes. In this chapter we are going to expand to 3D. Copy the equilateral triangle to the right onto a piece of paper and cut it out. Fold on the dotted lines. What shape do these four equilateral triangles make? If we place two of these equilateral triangles next to each other (like in the far right) what shape do these 8 equilateral triangles make?

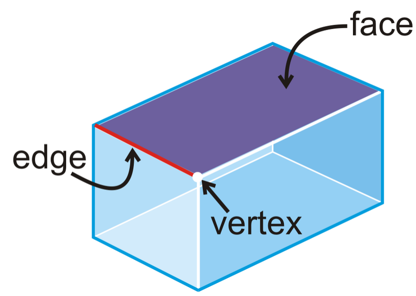




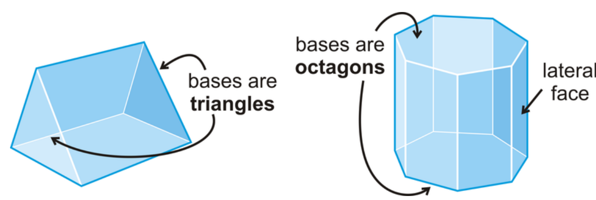
Polyhedrons

Polyhedron: A 3-dimensional figure that is formed by polygons that enclose a region in space.

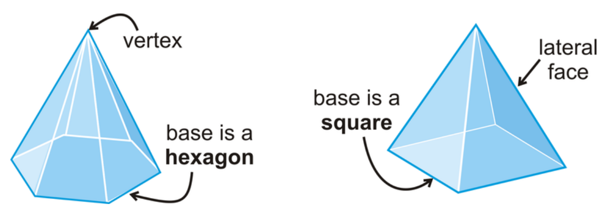
Each polygon in a polyhedron is called a ***face***. The line segment where two faces intersect is called an ***edge*** and the point of intersection of two edges is a ***vertex***. There are no gaps between the edges or vertices in a polyhedron. Examples of polyhedrons include a cube, prism, or pyramid. Non-polyhedrons are cones, spheres, and cylinders because they have sides that are not polygons.



Prism: A polyhedron with two congruent bases, in parallel planes, and the lateral sides are rectangles.



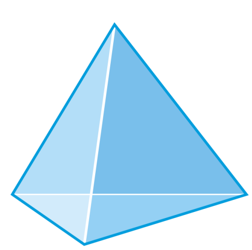
Pyramid: A polyhedron with one base and all the lateral sides meet at a common vertex. The lateral sides are triangles.



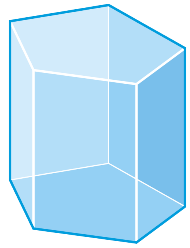
All prisms and pyramids are named by their bases. So, the first prism would be a triangular prism and the second would be an octagonal prism. The first pyramid would be a hexagonal pyramid and the second would be a square pyramid. The lateral faces of a pyramid are always triangles.

Example 1: Determine if the following solids are polyhedrons. If the solid is a polyhedron, name it and determine the number of faces, edges and vertices each has.

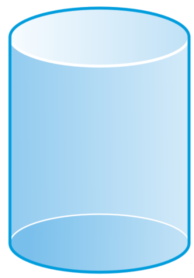
a)



b)



c)



Solution:

a) The base is a triangle and all the sides are triangles, so this is a polyhedron, a triangular pyramid. There are 4 faces, 6 edges and 4 vertices.

b) This solid is also a polyhedron because all the faces are polygons. The bases are both pentagons, so it is a pentagonal prism. There are 7 faces, 15 edges, and 10 vertices.

c) This is a cylinder and has bases that are circles. Circles are not polygons, so it is not a polyhedron.

Euler's Theorem

Let's put our results from Example 1 into a table.

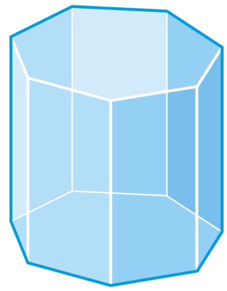
TABLE 11.1:

	Faces	*Vertices*	*Edges*
Triangular Pyramid	4	4	6
Pentagonal Prism	7	10	15

Notice that the sum of the faces + vertices is two more that the number of edges. This is called Euler's Theorem, after the Swiss mathematician Leonhard Euler.

Euler's Theorem: The number of faces (F), vertices (V), and edges (E) of a polyhedron can be related such that $F+V=E+2$.

Example 2: Find the number of faces, vertices, and edges in the octagonal prism.



Solution: Because this is a polyhedron, we can use Euler's Theorem to find either the number of faces, vertices or edges. It is easiest to count the faces, there are 10 faces. If we count the vertices, there are 16. Using this, we can solve for E in Euler's Theorem.

$$\begin{aligned} F+V &= E+2 \\ 10+16 &= E+2 \\ 24 &= E \qquad \text{There are 24 edges.} \end{aligned}$$

Example 3: In a six-faced polyhedron, there are 10 edges. How many vertices does the polyhedron have?

Solution: Solve for V in Euler's Theorem.

$$\begin{aligned} F+V &= E+2 \\ 6+V &= 10+2 \\ V &= 6 \qquad \text{There are 6 vertices.} \end{aligned}$$

Example 4: A three-dimensional figure has 10 vertices, 5 faces, and 12 edges. Is it a polyhedron?

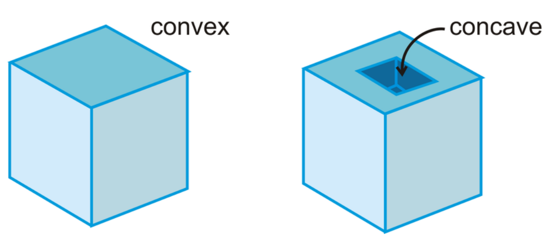
Solution: Plug in all three numbers into Euler's Theorem.

$$\begin{aligned} F+V &= E+2 \\ 5+10 &= 12+2 \\ 15 &\neq 14 \end{aligned}$$

Because the two sides are not equal, this figure is not a polyhedron.

Regular Polyhedra

Regular Polyhedron: A polyhedron where all the faces are congruent regular polygons.



Polyhedrons, just like polygons, can be ***convex*** or ***concave*** (also called non-convex). All regular polyhedron are convex. A concave polyhedron is similar to a concave polygon. The polyhedron "caves in," so that two non-adjacent vertices can be connected by a line segement that is outside the polyhedron.

There are five regular polyhedra called the Platonic solids, after the Greek philosopher Plato. These five solids are significant because they are the only five regular polyhedra. There are only five because the sum of the measures of the angles that meet at each vertex must be less than 360°. Therefore the only combinations are 3, 4 or 5 triangles at each vertex, 3 squares at each vertex or 3 pentagons. Each of these polyhedra have a name based on the number of sides, except the cube.

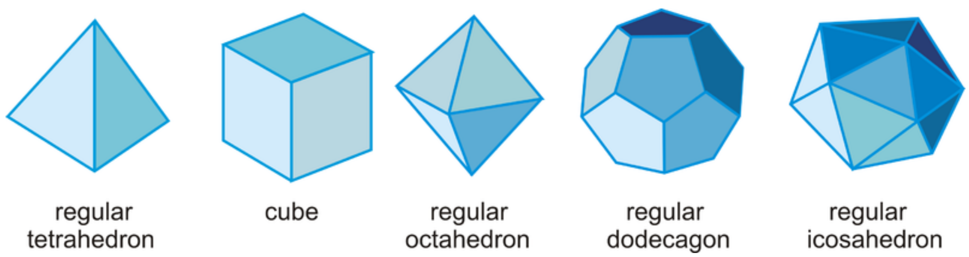
Regular Tetrahedron: A 4-faced polyhedron where all the faces are equilateral triangles.

Cube: A 6-faced polyhedron where all the faces are squares.

Regular Octahedron: An 8-faced polyhedron where all the faces are equilateral triangles.

Regular Dodecahedron: A 12-faced polyhedron where all the faces are regular pentagons.

Regular Icosahedron: A 20-faced polyhedron where all the faces are equilateral triangles.



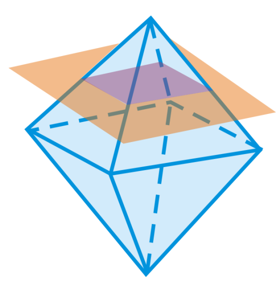
Cross-Sections

One way to "view" a three-dimensional figure in a two-dimensional plane, like this text, is to use cross-sections.

Cross-Section: The intersection of a plane with a solid.

Example 5: Describe the shape formed by the intersection of the plane and the regular octahedron.

a)



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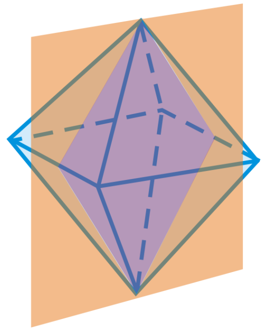
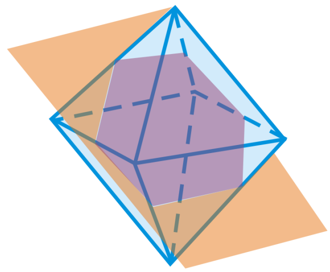


c)



Solution:

a) Square

b) Rhombus

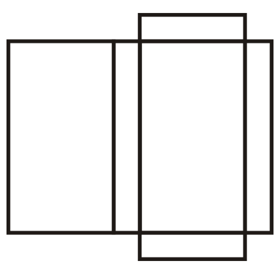
c) Hexagon

Nets

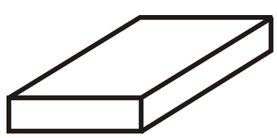
Another way to represent a three-dimensional figure in a two dimensional plane is to use a net.

Net: An unfolded, flat representation of the sides of a three-dimensional shape.

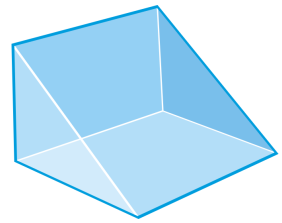
Example 6: What kind of figure does this net create?



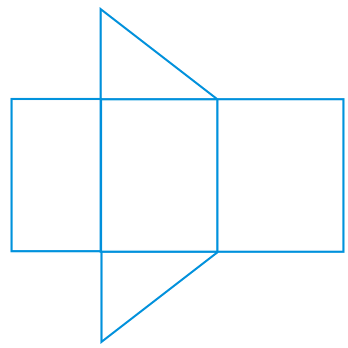
Solution: The net creates a rectangular prism.



Example 7: Draw a net of the right triangular prism below.



Solution: This net will have two triangles and three rectangles. The rectangles are all different sizes and the two triangles are congruent.



Notice that there could be a couple different interpretations of this, or any, net. For example, this net could have the triangles anywhere along the top or bottom of the three rectangles. Most prisms have multiple nets.

See the site http://www.cs.mcgill.ca/~sqrt/unfold/unfolding.html if you would like to see a few animations of other nets, including the Platonic solids.

Know What? Revisited The net of the first shape is a regular tetrahedron and the second is the net of a regular octahedron.

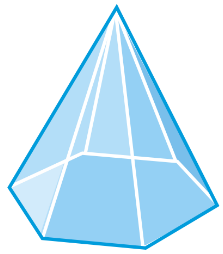
Review Questions

Complete the table using Euler's Theorem.

TABLE 11.2:

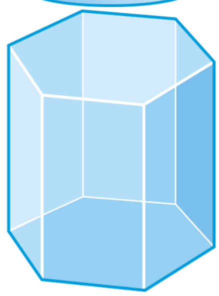
	Name	*Faces*	*Edges*	*Vertices*
1.	Rectangular Prism	6	12	
2.	Octagonal Pyramid		16	9
3.	Regular Icosahedron	20		12
4.	Cube		12	8
5.	Triangular Pyramid	4		4
6.	Octahedron	8	12	
7.	Heptagonal Prism		21	14
8.	Triangular Prism	5	9	

Determine if the following figures are polyhedra. If so, name the figure and find the number of faces, edges, and vertices.

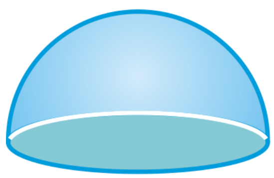


9.

10.



11.



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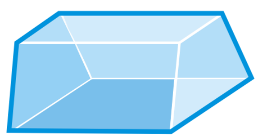

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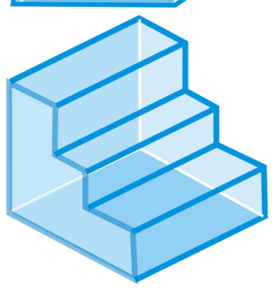
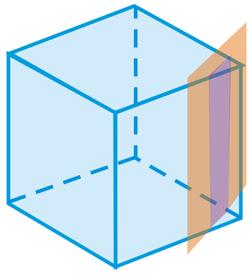
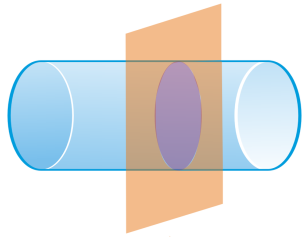


14.

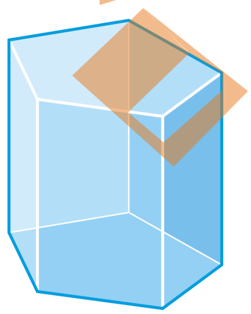
Describe the cross section formed by the intersection of the plane and the solid.



15.

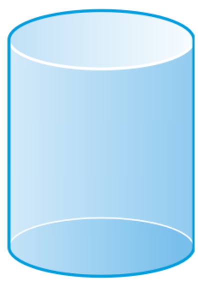


16.

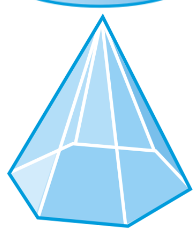


17.

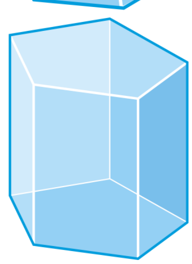
Draw the net for the following solids.



18.

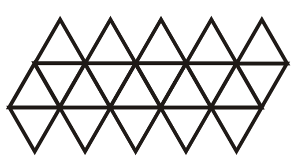


19.

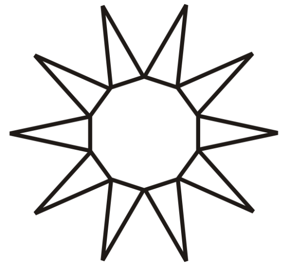


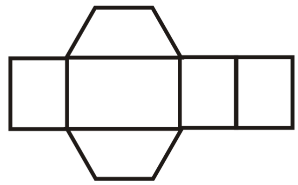
20.

Determine what shape is formed by the following nets.



21.

22. 

23. 

24. A cube has 11 unique nets. Draw 5 different nets of a cube.
25. A truncated icosahedron is a polyhedron with 12 regular pentagonal faces and 20 regular hexagonal faces and 90 edges. This icosahedron closely resembles a soccer ball. How many vertices does it have? Explain your reasoning.

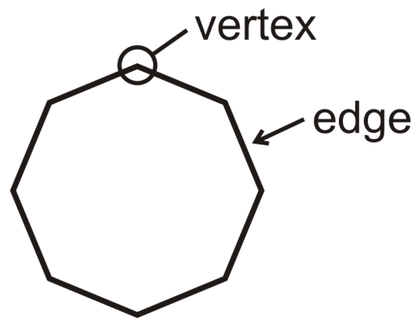
26. Use construction tools to construct a large equilateral triangle. Construct the three midsegments of the triangle. Cut out the equilateral triangle and fold along the midsegments. What net have you constructed?
27. Describe a method to construct a net for a regular octahedron.

For problems 28-30, we are going to connect the Platonic Solids to probability. A six sided die is the shape of a cube. The probability of any one side landing face up is $\frac{1}{6}$ because each of the six faces is congruent to each other.

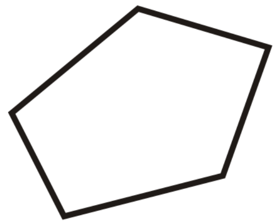
28. What shape would we make a die with 12 faces? If we number these faces 1 to 12, and each face has the same likelihood of landing face up, what is the probability of rolling a multiple of three?
29. I have a die that is a regular octahedron. Each face is labeled with a consecutive prime number starting with 2. What is the largest prime number on my die?
30. ***Challenge*** Rebecca wants to design a new die. She wants it to have one red face. The other faces will be yellow, blue or green. How many faces should her die have and how many of each color does it need so that: the probability of rolling yellow is eight times the probability of rolling red, the probability of rolling green is half the probability of rolling yellow and the probability of rolling blue is seven times the probability of rolling red?

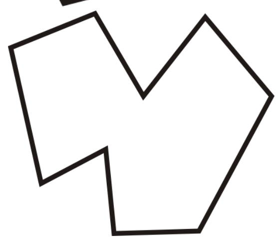
Review Queue Answers

a. There are 8 vertices and 8 edges in an octagon.



b. $5^2 = 25\ cm^2$
c. $\frac{1}{2} \cdot 10 \cdot 5\sqrt{3} = 25\sqrt{3}\ in^2$


a.


b.

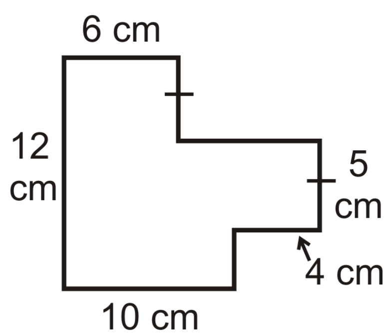
11.2 Surface Area of Prisms and Cylinders

Learning Objectives

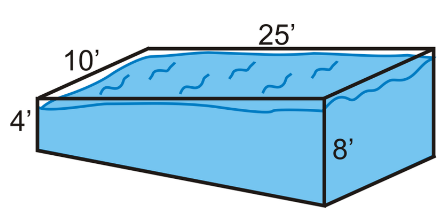
- Find the surface area of a prism.
- Find the surface area of a cylinder.

Review Queue

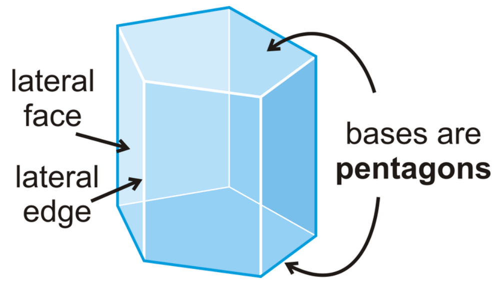
a. Find the area of a rectangle with sides:
   a. 6 and 9
   b. 11 and 4
   c. $5\sqrt{2}$ and $8\sqrt{6}$
b. If the area of a square is 36 $units^2$, what are the lengths of the sides?
c. If the area of a square is 45 $units^2$, what are the lengths of the sides?
d. Find the area of the shape. All sides are perpendicular. (Split the shape up into rectangles.)



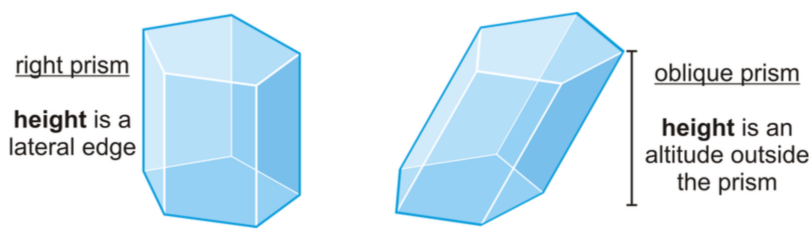
Know What? Your parents decide they want to put a pool in the backyard. They agree on a pool where the shallow end will be 4 ft. and the deep end will be 8 ft. The pool will be 10 ft. by 25 ft. How much siding do they need to buy to cover the sides and bottom of the pool? If the siding is \$25.00 a square yard, how much will it cost to enclose the pool?



Parts of a Prism



In the last section, we defined a prism as a 3-dimensional figure with 2 congruent bases, in parallel planes with rectangular lateral faces. The edges between the ***lateral faces*** are called ***lateral edges***. All prisms are named by their bases, so the prism to the right is a pentagonal prism. This particular prism is called a ***right prism*** because the lateral faces are perpendicular to the bases. ***Oblique prisms*** lean to one side or the other and the height is outside the prism.



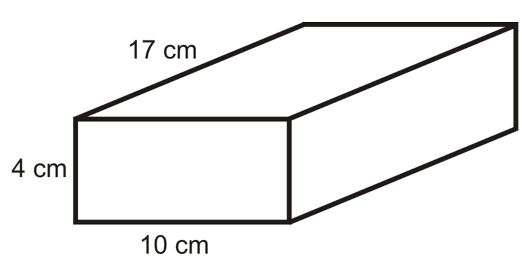
Surface Area of a Prism

Surface Area: The sum of the areas of the faces.

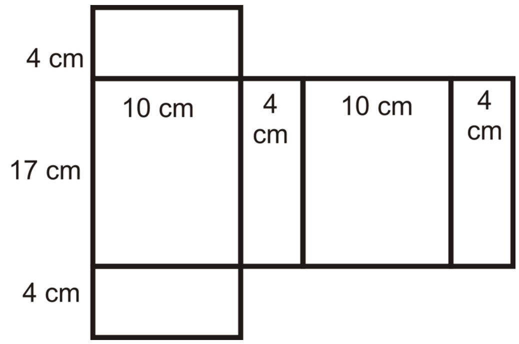
Lateral Area: The sum of the areas of the ***lateral*** faces.

You can use a net and the Area Addition Postulate to find the surface area of a right prism.

Example 1: Find the surface area of the prism below.



Solution: Open up the prism and draw the net. Determine the measurements for each rectangle in the net.



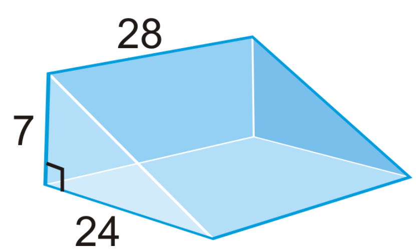
Using the net, we have:

$$\begin{aligned}SA_{prism} &= 2(4)(10)+2(10)(17)+2(17)(4)\\ &= 80+340+136\\ &= 556\ cm^2\end{aligned}$$

Because this is still area, the units are squared.

Surface Area of a Right Prism: The surface area of a right prism is the sum of the area of the bases and the area of each rectangular lateral face.

Example 2: Find the surface area of the prism below.

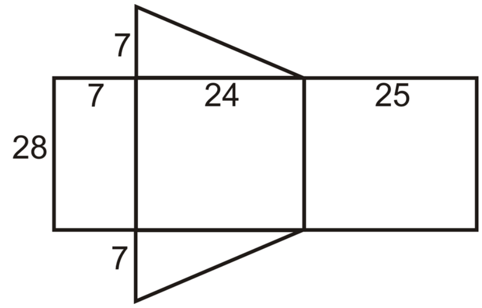


Solution: This is a right triangular prism. To find the surface area, we need to find the length of the hypotenuse of the base because it is the width of one of the lateral faces. Using the Pythagorean Theorem, the hypotenuse is

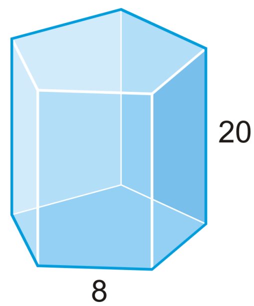
$$\begin{aligned}7^2+24^2 &= c^2\\ 49+576 &= c^2\\ 625 &= c^2\\ c &= 25\end{aligned}$$

Looking at the net, the surface area is:

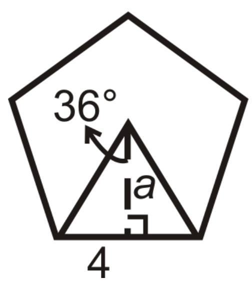
$$SA = 28(7)+28(24)+28(25)+2\left(\frac{1}{2}\cdot 7\cdot 24\right)$$
$$SA = 196+672+700+168 = 1736$$



Example 3: Find the surface area of the regular pentagonal prism.



Solution: For this prism, each lateral face has an area of 160 $units^2$. Then, we need to find the area of the regular pentagonal bases. Recall that the area of a regular polygon is $\frac{1}{2}asn$. $s = 8$ and $n = 5$, so we need to find a, the apothem.

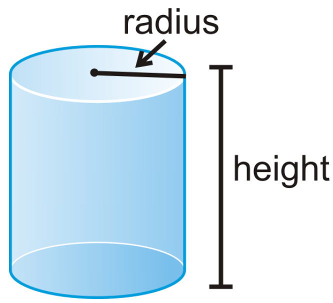


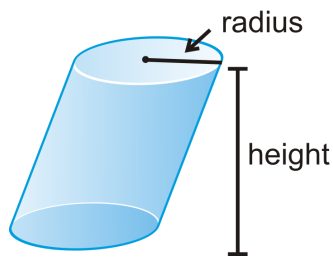
$$\tan 36^\circ = \frac{4}{a}$$
$$a = \frac{4}{\tan 36^\circ} \approx 5.51$$
$$SA = 5(160) + 2\left(\frac{1}{2} \cdot 5.51 \cdot 8 \cdot 5\right) = 1020.4$$

Cylinders

Cylinder: A solid with congruent circular bases that are in parallel planes. The space between the circles is enclosed.

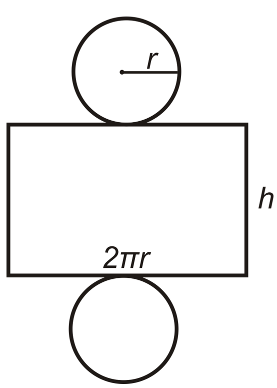
Just like a circle, the cylinder has a radius for each of the circular bases. Also, like a prism, a cylinder can be oblique, like the one to the right.





Surface Area of a Right Cylinder

Let's find the net of a right cylinder. One way for you to do this is to take a label off of a soup can or can of vegetables. When you take this label off, we see that it is a rectangle where the height is the height of the cylinder and the base is the circumference of the base. This rectangle and the two circular bases make up the net of a cylinder.



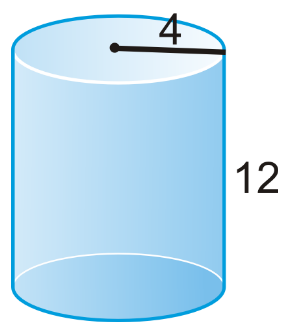
From the net, we can see that the surface area of a right cylinder is

$$\underbrace{2\pi r^2}_{\substack{\text{area of}\\ \text{both}\\ \text{circles}}} + \underbrace{2\pi r}_{\substack{\text{length}\\ \text{of}\\ \text{rectangle}}} h$$

Surface Area of a Right Cylinder: If r is the radius of the base and h is the height of the cylinder, then the surface area is $SA = 2\pi r^2 + 2\pi rh$.

To see an animation of the surface area, click http://www.rkm.com.au/ANIMATIONS/animation-Cylinder-Surface-Area-Derivation.html , by Russell Knightley.

Example 4: Find the surface area of the cylinder.



Solution: $r = 4$ and $h = 12$. Plug these into the formula.

$$\begin{aligned} SA &= 2\pi(4)^2 + 2\pi(4)(12) \\ &= 32\pi + 96\pi \\ &= 128\pi \end{aligned}$$

Example 5: The circumference of the base of a cylinder is 16π and the height is 21. Find the surface area of the cylinder.

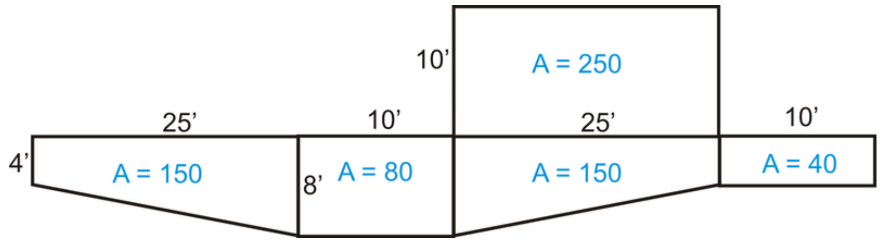
Solution: If the circumference of the base is 16π, then we can solve for the radius.

$$\begin{aligned} 2\pi r &= 16\pi \\ r &= 8 \end{aligned}$$

Now, we can find the surface area.

$$\begin{aligned} SA &= 2\pi(8)^2 + (16\pi)(21) \\ &= 128\pi + 336\pi \\ &= 464\pi \end{aligned}$$

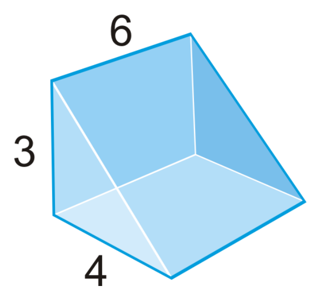
Know What? Revisited To the right is the net of the pool (minus the top). From this, we can see that your parents would need 670 square feet of siding. This means that the total cost would be $5583.33 for the siding.



Review Questions

1. How many square feet are in a square yard?
2. How many square centimeters are in a square meter?

Use the right triangular prism to answer questions 3-6.



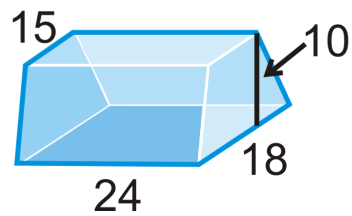
3. What shape are the bases of this prism? What are their areas?
4. What are the dimensions of each of the lateral faces? What are their areas?
5. Find the lateral surface area of the prism.
6. Find the total surface area of the prism.
7. ***Writing*** Describe the difference between lateral surface area and total surface area.
8. The lateral surface area of a cylinder is what shape? What is the area of this shape?
9. Fuzzy dice are cubes with 4 inch sides.

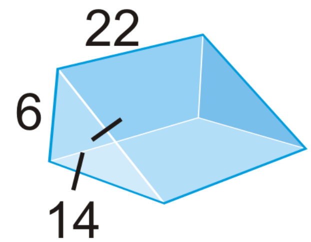
   a. What is the surface area of one die?
   b. Typically, the dice are sold in pairs. What is the surface area of two dice?

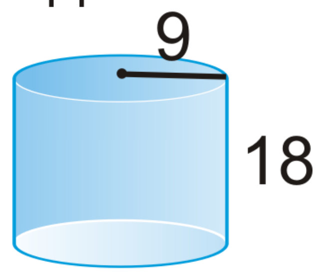
10. A right cylinder has a 7 cm radius and a height of 18 cm. Find the surface area.

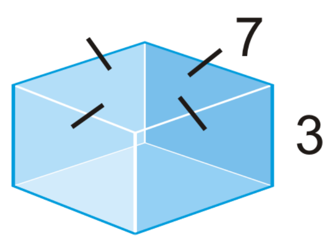
Find the surface area of the following solids. Leave answers in terms of π.

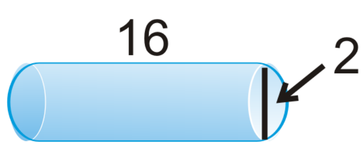
11. bases are isosceles trapezoids

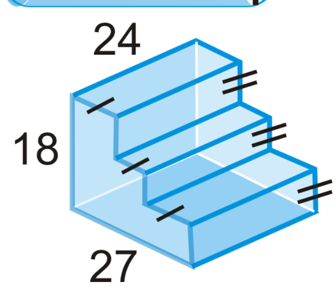


12. 

13. 

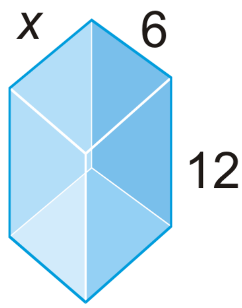
14. 

15. 

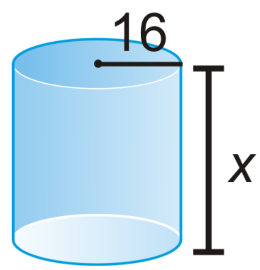
16. 

Algebra Connection Find the value of x, given the surface area.

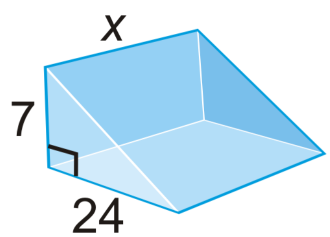
17. $SA = 432\ units^2$



18. $SA = 1536\pi$ $units^2$

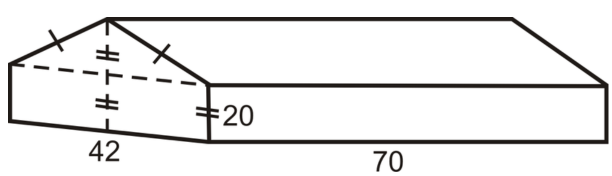


19. $SA = 1568$ $units^2$

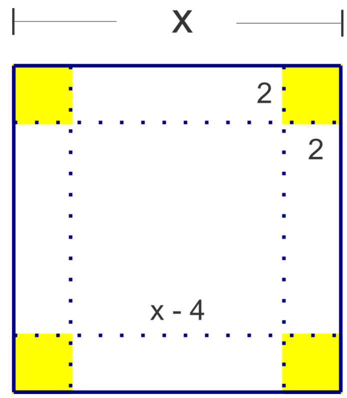


20. The area of the base of a cylinder is 25π in^2 and the height is 6 in. Find the *lateral* surface area.
21. The circumference of the base of a cylinder is 80π cm and the height is 36 cm. Find the total surface area.
22. The lateral surface area of a cylinder is 30π m^2. What is one possibility for height of the cylinder?

Use the diagram below for questions 23-27. The barn is shaped like a pentagonal prism with dimensions shown in feet.



23. What is the area of the roof? (Both sides)
24. What is the floor area of the barn?
25. What is the area of the sides of the barn?
26. The farmer wants to paint the sides of the roof (excluding the roof). If a gallon of paint covers 250 square feet, how many gallons will he need?
27. A gallon of paint costs \$15.50. How much will it cost for him to paint the sides of the barn?
28. Charlie started a business canning artichokes. His cans are 5 in tall and have diameter 4 in. If the label must cover the entire lateral surface of the can and the ends must overlap by at least one inch, what are the dimensions and area of the label?
29. An open top box is made by cutting out 2 in by 2 in squares from the corners of a large square piece of cardboard. Using the picture as a guide, find an expression for the surface area of the box. If the surface area is 609 in^2, find the length of x. Remember, there is no top.



30. Find an expression for the surface area of a cylinder in which the ratio of the height to the diameter is 2:1. If x is the diameter, use your expression to find x if the surface area is 160π.

Review Queue Answers

a.
   a. 54
   b. 44
   c. $80\sqrt{3}$
b. $s = 6$
c. $s = 3\sqrt{5}$
d. $A = 60 + 30 + 20 = 110\ cm^2$

11.3 Surface Area of Pyramids and Cones

Learning Objectives

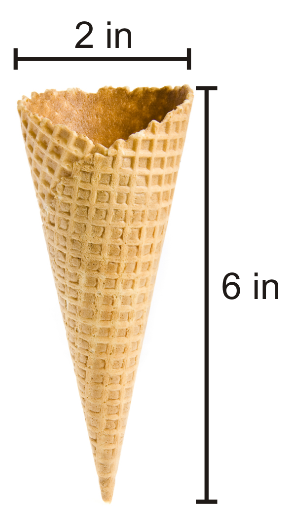
- Find the surface area of a pyramid.
- Find the surface area of a cone.

Review Queue

a. A rectangular prism has sides of 5 cm, 6 cm, and 7 cm. What is the surface area?
b. Triple the dimensions of the rectangular prism from #1. What is its surface area?
c. A cylinder has a diameter of 10 in and a height of 25 in. What is the surface area?
d. A cylinder has a circumference of $72\pi\ ft$. and a height of 24 ft. What is the surface area?
e. Draw the net of a square pyramid.

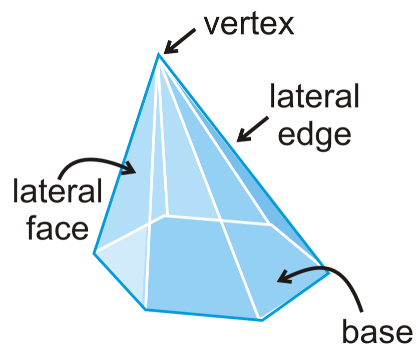
Know What? A typical waffle cone is 6 inches tall and has a diameter of 2 inches. This happens to be your friend Jeff's favorite part of his ice cream dessert. You decide to use your mathematical prowess to figure out exactly how much waffle cone Jeff is eating. What is the surface area of the waffle cone? (You may assume that the cone is straight across at the top)

Jeff decides he wants a "king size" cone, which is 8 inches tall and has a diameter of 4 inches. What is the surface area of this cone?

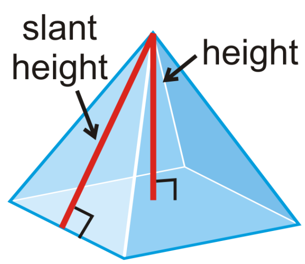


Parts of a Pyramid

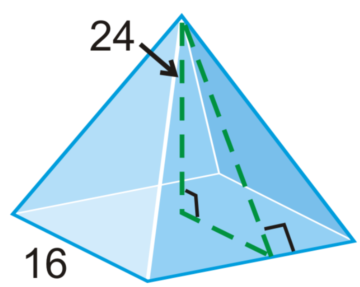
A pyramid has one ***base*** and all the ***lateral faces*** meet at a common ***vertex***. The edges between the lateral faces are ***lateral edges***. The edges between the base and the lateral faces are called ***base edges***. If we were to draw the height of the pyramid to the right, it would be off to the left side.



When a pyramid has a height that is directly in the center of the base, the pyramid is said to be regular. These pyramids have a regular polygon as the base. All regular pyramids also have a ***slant height*** that is the height of a lateral face. Because of the nature of regular pyramids, ***all slant heights are congruent***. A non-regular pyramid does not have a slant height.



Example 1: Find the slant height of the square pyramid.



Solution: Notice that the slant height is the hypotenuse of a right triangle formed by the height and half the base length. Use the Pythagorean Theorem.

$$\begin{aligned} 8^2+24^2 &= l^2 \\ 64+576 &= l^2 \\ 640 &= l^2 \\ l &= \sqrt{640} = 8\sqrt{10} \end{aligned}$$

Surface Area of a Regular Pyramid

Using the slant height, which is usually labeled l, the area of each triangular face is $A = \frac{1}{2}bl$.

Example 2: Find the surface area of the pyramid from Example 1.

Solution: The surface area of the four triangular faces are $4\left(\frac{1}{2}bl\right) = 2(16)\left(8\sqrt{10}\right) = 256\sqrt{10}$. To find the total surface area, we also need the area of the base, which is $16^2 = 256$. The total surface area is $256\sqrt{10} + 256 \approx 1065.54$.

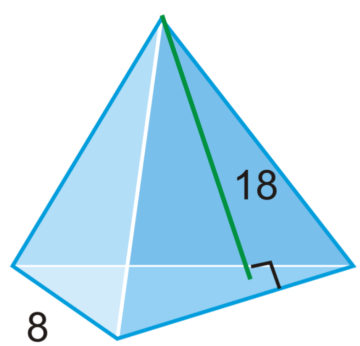
From this example, we see that the formula for a square pyramid is:

$$SA = (area\ of\ the\ base) + 4(area\ of\ triangular\ faces)$$
$$SA = B + n\left(\frac{1}{2}bl\right) \qquad B \text{ is the area of the base and } n \text{ is the number of triangles.}$$
$$SA = B + \frac{1}{2}l(nb) \qquad \text{Rearranging the variables, } nb = P, \text{ the perimeter of the base.}$$
$$SA = B + \frac{1}{2}Pl$$

Surface Area of a Regular Pyramid: If B is the area of the base and P is the perimeter of the base and l is the slant height, then $SA = B + \frac{1}{2}Pl$.

If you ever forget this formula, use the net. Each triangular face is congruent, plus the area of the base. This way, you do not have to remember a formula, just a process, which is the same as finding the area of a prism.

Example 3: Find the area of the regular triangular pyramid.



Solution: The area of the base is $A = \frac{1}{4}s^2\sqrt{3}$ because it is an equilateral triangle.

$$B = \frac{1}{4}8^2\sqrt{3} = 16\sqrt{3}$$
$$SA = 16\sqrt{3} + \frac{1}{2}(24)(18) = 16\sqrt{3} + 216 \approx 243.71$$

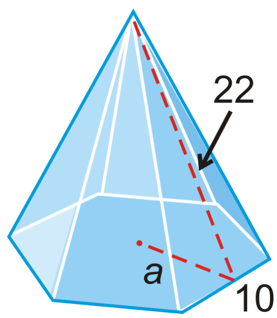
Example 4: If the lateral surface area of a square pyramid is $72\ ft^2$ and the base edge is equal to the slant height, what is the length of the base edge?

Solution: In the formula for surface area, the lateral surface area is $\frac{1}{2}Pl$ or $\frac{1}{2}nbl$. We know that $n = 4$ and $b = l$. Let's solve for b.

$$\frac{1}{2}nbl = 72\ ft^2$$
$$\frac{1}{2}(4)b^2 = 72$$
$$2b^2 = 72$$
$$b^2 = 36$$
$$b = 6$$

Therefore, the base edges are all 6 units and the slant height is also 6 units.

Example 4: Find the area of the regular hexagonal pyramid below.

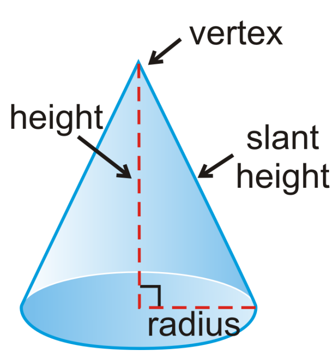


Solution: To find the area of the base, we need to find the apothem. If the base edges are 10 units, then the apothem is $5\sqrt{3}$ for a regular hexagon. The area of the base is $\frac{1}{2}asn = \frac{1}{2}\left(5\sqrt{3}\right)(10)(6) = 150\sqrt{3}$. The total surface area is:

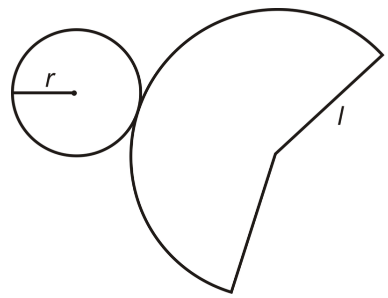
$$SA = 150\sqrt{3} + \frac{1}{2}(6)(10)(22)$$
$$= 150\sqrt{3} + 660 \approx 919.81\ units^2$$

Surface Area of a Cone

Cone: A solid with a circular base and sides taper up towards a common vertex.



It is said that a cone is generated from rotating a right triangle around one leg in a circle. Notice that a cone has a slant height, just like a pyramid. The surface area of a cone is a little trickier, however. We know that the base is a circle, but we need to find the formula for the curved side that tapers up from the base. Unfolding a cone, we have the net:



From this, we can see that the lateral face's edge is $2\pi r$ and the sector of a circle with radius l. We can find the area of the sector by setting up a proportion.

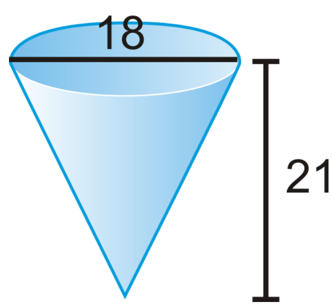
$$\frac{Area\ of\ circle}{Area\ of\ sector} = \frac{Circumference}{Arc\ length}$$
$$\frac{\pi l^2}{Area\ of\ sector} = \frac{2\pi l}{2\pi r} = \frac{l}{r}$$

Cross multiply:

$$l(Area\ of\ sector) = \pi r l^2$$
$$Area\ of\ sector = \pi r l$$

Surface Area of a Right Cone: The surface area of a right cone with slant height l and base radius r is $SA = \pi r^2 + \pi r l$.

Example 5: What is the surface area of the cone?



Solution: In order to find the surface area, we need to find the slant height. Recall from a pyramid, that the slant height forms a right triangle with the height and the radius. Use the Pythagorean Theorem.

$$l^2 = 9^2 + 21^2$$
$$= 81 + 441$$
$$l = \sqrt{522} \approx 22.85$$

The surface area would be $SA = \pi 9^2 + \pi(9)(22.85) \approx 900.54\ units^2$.

Example 6: The surface area of a cone is 36π and the slant height is 5 units. What is the radius?

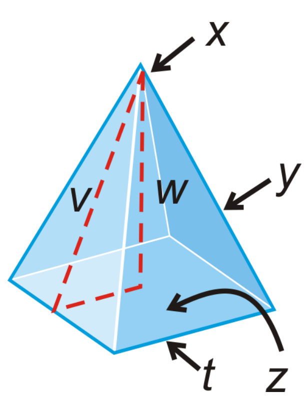
Solution: Plug in what you know into the formula for the surface area of a cone and solve for r.

$$\begin{aligned}
36\pi &= \pi r^2 + \pi r(5) && \text{Because every term has } \pi, \text{ we can cancel it out.}\\
36 &= r^2 + 5r && \text{Set one side equal to zero, and this becomes a factoring problem.}\\
r^2 + 5r - 36 &= 0 &&\\
(r-4)(r+9) &= 0 && \text{The possible answers for } r \text{ are 4 and } -9. \text{ The radius must be positive,}\\
& && \text{so our answer is 4.}
\end{aligned}$$

Know What? Revisited The standard cone has a surface area of $\pi + 6\pi = 7\pi \approx 21.99\ in^2$. The "king size" cone has a surface area of $4\pi + 16\pi = 20\pi \approx 62.83$, almost three times as large as the standard cone.

Review Questions

Fill in the blanks about the diagram to the left.

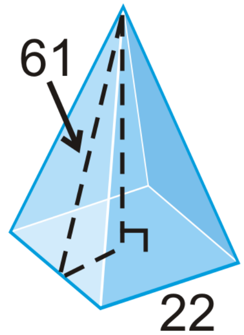


1. x is the ___________.
2. The slant height is ________.
3. y is the ___________.
4. The height is ________.
5. The base is _______.
6. The base edge is ________.
7. Sketch a right cone. Label the *height, slant height*, and *radius*.

For questions 8-10, sketch each of the following solids and answer the question. Your drawings should be to scale, but not one-to-one. Leave your answer in simplest radical form.

8. Draw a right cone with a radius of 5 cm and a height of 15 cm. What is the slant height?
9. Draw a square pyramid with an edge length of 9 in and a 12 in height. Find the slant height.
10. Draw an equilateral triangle pyramid with an edge length of 6 cm and a height of 6 cm. ***Describe*** how you would find the slant height and then find it.

Find the area of a lateral face of the regular pyramid. Leave your answer in simplest radical form.



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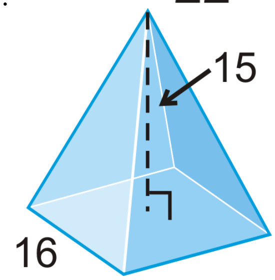

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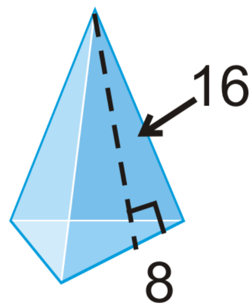
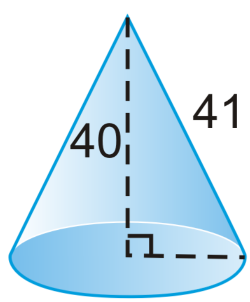


13.

Find the surface area of the regular pyramids and right cones. Round your answers to 2 decimal places.



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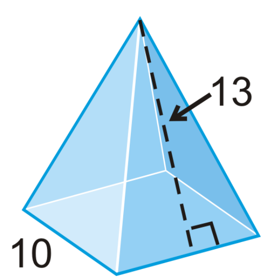

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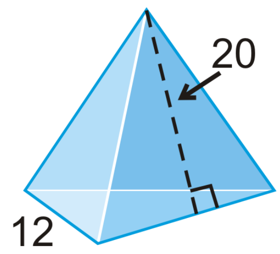
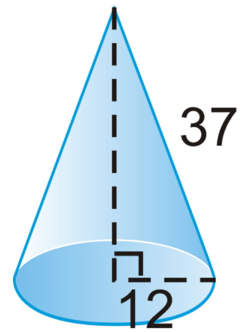
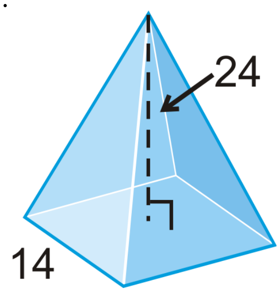
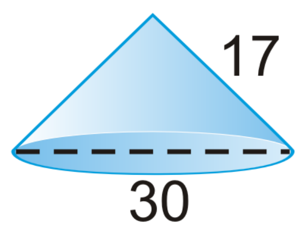
16. 

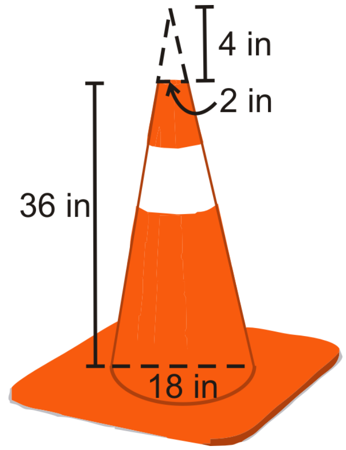
17. 

18. 

19. 

20. From these pictures, we see that a regular triangle pyramid does not have to have four congruent faces. How many faces must be congruent?
21. A ***regular tetrahedron*** has four equilateral triangles as its faces. Find the surface area of a regular tetrahedron with edge length of 6 units.
22. Using the formula for the area of an equilateral triangle, what is the surface area of a regular tetrahedron, with edge length s?

Challenge Find the surface area of the traffic cone with the given information. The gone is cut off at the top (4 inch cone) and the base is a square with sides of length 24 inches. Round answers to the nearest hundredth.



23. Find the area of the entire square. Then, subtract the area of the base of the cone.
24. Find the lateral area of the cone portion (include the 4 inch cut off top of the cone).
25. Now, subtract the cut-off top of the cone, to only have the lateral area of the cone portion of the traffic cone.
26. Combine your answers from #23 and #25 to find the entire surface area of the traffic cone.

For questions 27-30, consider the sector of a circle with radius 25 cm and arc length 14π.

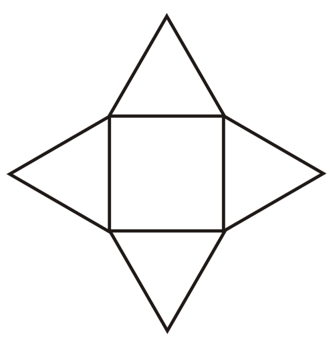
27. What is the central angle of this sector?
28. If this sector is rolled into a cone, what are the radius and area of the base of the cone?
29. What is the height of this cone?
30. What is the total surface are of the cone?

For questions 31-33, consider a square with diagonal length $10\sqrt{2}\ in$.

31. What is the length of a side of the square?
32. If this square is the base of a right pyramid with height 12, what is the slant height of the pyramid?
33. What is the surface area of the pyramid?

Review Queue Answers

a. $2(5\cdot 6)+2(5\cdot 7)+2(6\cdot 7) = 214\ cm^2$
b. $2(15\cdot 18)+2(15\cdot 21)+2(18\cdot 21) = 1926\ cm^2$
c. $2\cdot 25\pi + 250\pi = 300\pi\ in^2$
d. $36^2(2\pi)+72\pi(24) = 4320\pi\ ft^2$
e.



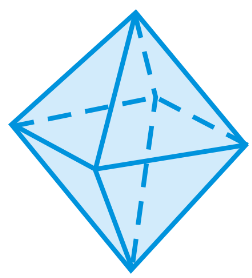
11.4 Volume of Prisms and Cylinders

Learning Objectives

- Find the volume of a prism.
- Find the volume of a cylinder.

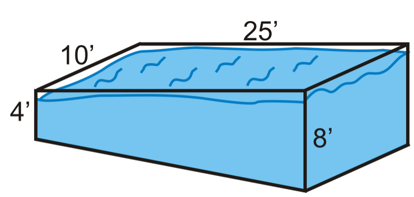
Review Queue

a. Define volume in your own words.
b. What is the surface area of a cube with 3 inch sides?
c. What is the surface area of a cube with $4\sqrt{2}$ inch sides?
d. A regular octahedron has 8 congruent equilateral triangles as the faces.



a. If each edge is 4 cm, what is the surface area of the figure?
b. If each edge is s, what is the surface area of the figure?

Know What? The pool is done and your family is ready to fill it with water. Recall that the shallow end is 4 ft. and the deep end is 8 ft. The pool is 10 ft. wide by 25 ft. long. How many gallons of water will it take to fill the pool? There are approximately 7.48 gallons in a cubic foot.



Volume of a Rectangular Prism

Volume: The measure of how much space a three-dimensional figure occupies.

Another way to define volume would be how much a three-dimensional figure can hold, water or sand, for example. The basic unit of volume is the cubic unit: cubic centimeter (cm^3), cubic inch (in^3), cubic meter (m^3), cubic foot (ft^3), etc. Each basic cubic unit has a measure of one for each: length, width, and height.

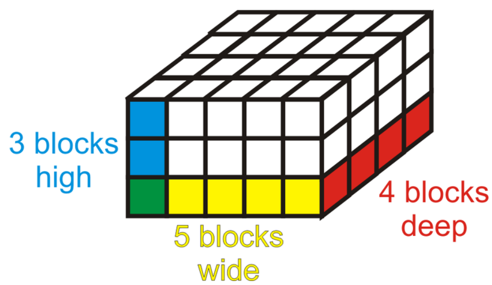
Volume of a Cube Postulate: The volume of a cube is the cube of the length of its side, or s^3.

What this postulate tells us is that every solid can be broken down into cubes, going along with our basic unit of measurement, the cubic unit. For example, if we wanted to find the volume of a cube with one inch sides, it would be $1^3 = 1\ in^3$. If we wanted to find the volume of a cube with 9 inch sides, it would be $9^3 = 729\ in^3$.

Volume Congruence Postulate: If two solids are congruent, then their volumes are congruent.

Volume Addition Postulate: The volume of a solid is the sum of the volumes of all of its non-overlapping parts.

Example 1: Find the volume of the right rectangular prism below.



Solution: A rectangular prism can be made from any square cubes. To find the volume, we would simply count the cubes. The bottom layer has 20 cubes, or 4 times 5, and there are 3 layers, or the same as the height. Therefore there are 60 cubes in this prism and the volume would be 60 $units^3$.

But, what if we didn't have cubes? Let's generalize this formula for any rectangular prism. Notice that each layer is the same as the area of the base. Then, we multiplied by the height. Here is our formula.

Volume of a Rectangular Prism: If a rectangular prism is h units high, w units wide, and l units long, then its volume is $V = l \cdot w \cdot h$.

Example 2: A typical shoe box is 8 in by 14 in by 6 in. What is the volume of the box?

Solution: We can assume that a shoe box is a rectangular prism. Therefore, we can use the formula above.

$$V = (8)(14)(6) = 672\ in^2$$

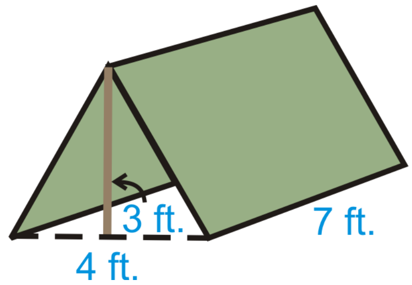
Volume of any Prism

If we further analyze the formula for the volume of a rectangular prism, we would see that $l \cdot w$ is equal to the area of the base of the prism, a rectangle. If the bases are not rectangles, this would still be true, however we would have to rewrite the equation a little.

Volume of a Prism: If the area of the base of a prism is B and the height is h, then the volume is $V = B \cdot h$.

Notice that "B" is not always going to be the same. So, to find the volume of a prism, you would first find the area of the base and then multiply it by the height.

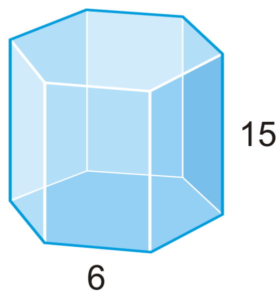
Example 3: You have a small, triangular prism shaped tent. How much volume does it have, once it is set up?



Solution: First, we need to find the area of the base. That is going to be $B = \frac{1}{2}(3)(4) = 6\ ft^2$. Multiplying this by 7 we would get the entire volume. The volume is $42\ ft^3$.

Even though the height in this problem does not look like a "height," it is, when referencing the formula. Usually, the height of a prism is going to be the last length you need to use.

Example 4: Find the volume of the regular hexagonal prism below.



Solution: Recall that a regular hexagon is divided up into six equilateral triangles. The height of one of those triangles would be the apothem. If each side is 6, then half of that is 3 and half of an equilateral triangle is a 30-60-90 triangle. Therefore, the apothem is going to be $3\sqrt{3}$. The area of the base is:

$$B = \frac{1}{2}\left(3\sqrt{3}\right)(6)(6) = 54\sqrt{3}\ units^2$$

And the volume will be:

$$V = Bh = \left(54\sqrt{3}\right)(15) = 810\sqrt{3}\ units^3$$

Cavalieri's Principle

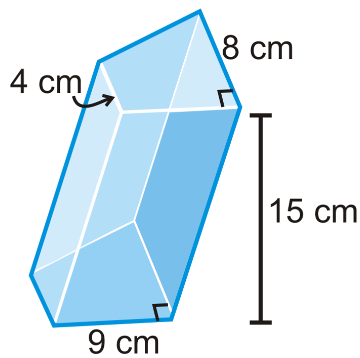
Recall that earlier in this section we talked about oblique prisms. These are prisms that lean to one side and the height is outside the prism. What would be the area of an oblique prism? To answer this question, we need to introduce Cavalieri's Principle. Consider to piles of books below.

Both piles have 15 books, therefore they will have the same volume. However, one pile is leaning. Cavalieri's Principle says that this does not matter, as long as the heights are the same and every horizontal cross section has the same area as the base, the volumes are the same.

Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.

Basically, if an oblique prism and a right prism have the same base area and height, then they will have the same volume.

Example 5: Find the area of the oblique prism below.



Solution: This is an oblique right trapezoidal prism. First, find the area of the trapezoid.

$$B = \frac{1}{2}(9)(8+4) = 9(6) = 54\ cm^2$$

Then, multiply this by the height.

$$V = 54(15) = 810\ cm^3$$

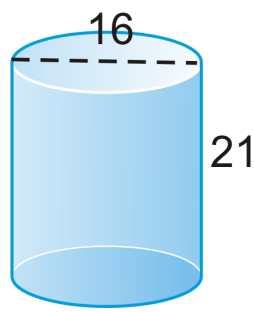
Volume of a Cylinder

If we use the formula for the volume of a prism, $V = Bh$, we can find the volume of a cylinder. In the case of a cylinder, the base, or B, would be the area of a circle. Therefore, the volume of a cylinder would be $V = (\pi r^2)h$, where πr^2 is the area of the base.

Volume of a Cylinder: If the height of a cylinder is h and the radius is r, then the volume would be $V = \pi r^2 h$.

Also, like a prism, Cavalieri's Principle holds. So, the volumes of an oblique cylinder and a right cylinder have the same formula.

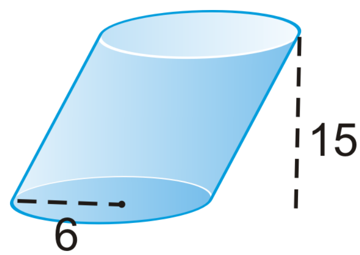
Example 6: Find the volume of the cylinder.



Solution: If the diameter is 16, then the radius is 8.

$$V = \pi 8^2(21) = 1344\pi \ units^3$$

Example 7: Find the volume of the cylinder.



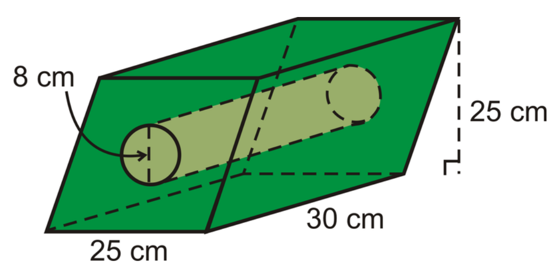
Solution: $V = \pi 6^2(15) = 540\pi \ units^3$

Example 8: If the volume of a cylinder is $484\pi \ in^3$ and the height is 4 in, what is the radius?

Solution: Plug in what you know to the volume formula and solve for r.

$$\begin{aligned} 484\pi &= \pi r^2(4) \\ 121 &= r^2 \\ 11 &= r \end{aligned}$$

Example 9: Find the volume of the solid below.



Solution: This solid is a parallelogram-based prism with a cylinder cut out of the middle. To find the volume, we need to find the volume of the prism and then subtract the volume of the cylinder.

$$\begin{aligned} V_{prism} &= (25 \cdot 25)30 = 18750 \ cm^3 \\ V_{cylinder} &= \pi(4)^2(30) = 480\pi \ cm^3 \end{aligned}$$

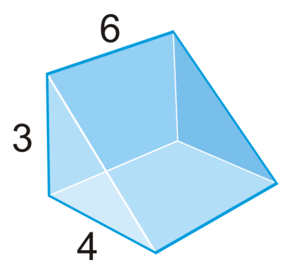
The total volume is $18750 - 480\pi \approx 17242.04\ cm^3$.

Know What? Revisited Even though it doesn't look like it, the trapezoid is considered the base of this prism. The area of the trapezoids are $\frac{1}{2}(4+8)25 = 150\ ft^2$. Multiply this by the height, 10 ft, and we have that the volume is $1500\ ft^3$. To determine the number of gallons that are needed, divide 1500 by 7.48. $\frac{1500}{7.48} \approx 200.53$ gallons are needed to fill the pool.

Review Questions

1. Two cylinders have the same surface area. Do they have the same volume? How do you know?
2. How many one-inch cubes can fit into a box that is 8 inches wide, 10 inches long, and 12 inches tall? Is this the same as the volume of the box?
3. A cereal box in 2 inches wide, 10 inches long and 14 inches tall. How much cereal does the box hold?
4. A can of soda is 4 inches tall and has a diameter of 2 inches. How much soda does the can hold? Round your answer to the nearest hundredth.
5. A cube holds 216 in^3. What is the length of each edge?
6. A cylinder has a volume of $486\pi\ ft.^3$. If the height is 6 ft., what is the diameter?

Use the right triangular prism to answer questions 7 and 8.

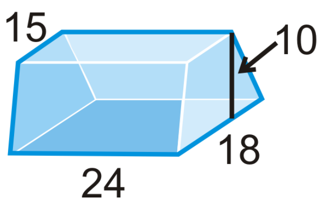


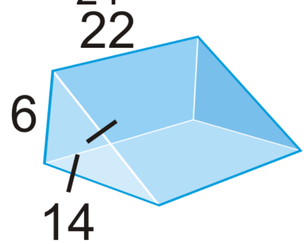
7. What is the length of the third base edge?
8. Find the volume of the prism.
9. Fuzzy dice are cubes with 4 inch sides.

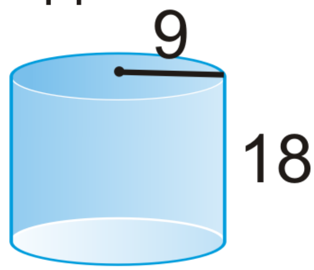
   a. What is the volume of one die?
   b. What is the volume of both dice?

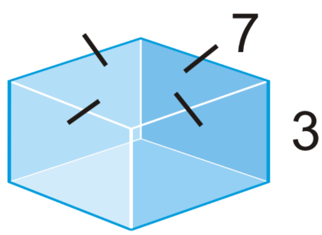
10. A right cylinder has a 7 cm radius and a height of 18 cm. Find the volume.

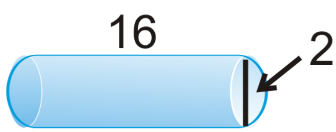
Find the volume of the following solids. Round your answers to the nearest hundredth.

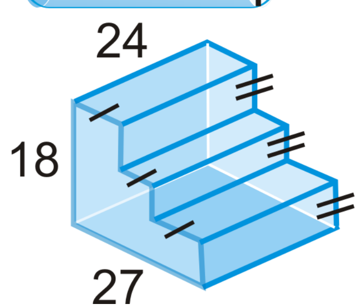
11. 

12. 

13. 

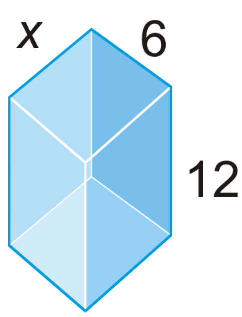
14. 

15. 

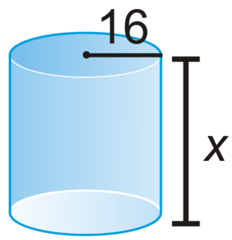
16. 

Algebra Connection Find the value of x, given the surface area.

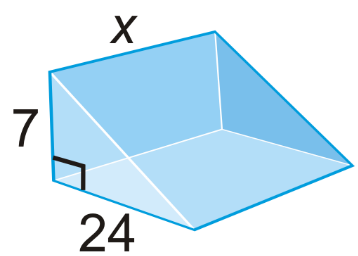
17. $V = 504\ units^3$



18. $V = 6144\pi\ units^3$

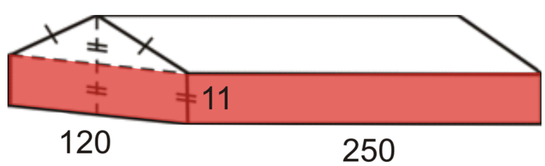


19. $V = 2688\ units^3$



20. The area of the base of a cylinder is $49\pi\ in^2$ and the height is 6 in. Find the volume.
21. The circumference of the base of a cylinder is $80\pi\ cm$ and the height is 15 cm. Find the volume.
22. The lateral surface area of a cylinder is $30\pi\ m^2$ and the circumference is $10\pi\ m$. What is the volume of the cylinder?

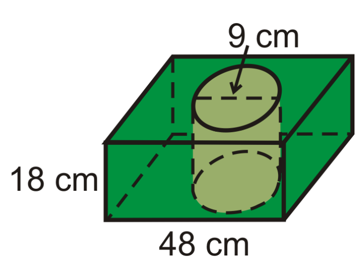
Use the diagram below for questions 23-25. The barn is shaped like a pentagonal prism with dimensions shown in feet.

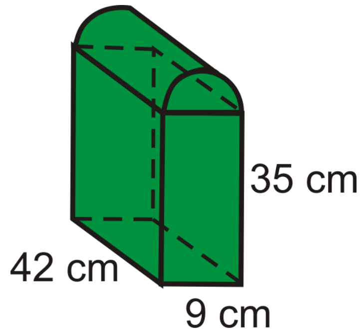


23. Find the volume of the red rectangular prism.
24. Find the volume of the triangular prism on top of the rectangular prism.
25. Find the total volume of the barn.

Find the volume of the composite solids below. Round your answers to the nearest hundredth.

26. The bases are squares.



27. 

28. The volume of a cylinder with height to radius ratio of 4:1 is $108\pi\ cm^3$. Find the radius and height of the cylinder.
29. The length of a side of the base of a hexagonal prism is 8 cm and its volume is $1056\sqrt{3}\ cm^3$. Find the height of the prism.
30. A cylinder fits tightly inside a rectangular prism with dimensions in the ratio 5:5:7 and volume $1400\ in^3$. Find the volume of the space inside the prism that is not contained in the cylinder.

Review Queue Answers

a. The amount a three-dimensional figure can hold.
b. $54\ in^2$
c. $192\ in^2$
   a. $8\left(\frac{1}{4}\cdot 4^2\sqrt{3}\right) = 32\sqrt{3}\ cm^2$
   b. $8\left(\frac{1}{4}\cdot s^2\sqrt{3}\right) = 2s^2\sqrt{3}$

11.5 Volume of Pyramids and Cones

Learning Objectives

- Find the volume of a pyramid.
- Find the volume of a cone.

Review Queue

a. Find the volume of a square prism with 8 inch base edges and a 12 inch height.
b. Find the volume of a cylinder with a *diameter* of 8 inches and a height of 12 inches.
c. In your answers from #1 and #2, which volume is bigger? Why do you think that is?
d. Find the surface area of a square pyramid with 10 inch base edges and a height of 12 inches.

Know What? The Khafre Pyramid is the second largest pyramid of the Ancient Egyptian Pyramids in Giza. It is a square pyramid with a base edge of 706 feet and an original height of 407.5 feet. What was the original volume of the Khafre Pyramid?

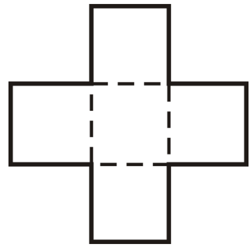
Volume of a Pyramid

Recall that the volume of a prism is Bh, where B is the area of the base. The volume of a pyramid is closely related to the volume of a prism with the same sized base.

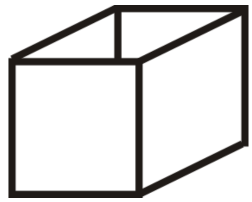
Investigation 11-1: Finding the Volume of a Pyramid

Tools needed: pencil, paper, scissors, tape, ruler, dry rice or sand.

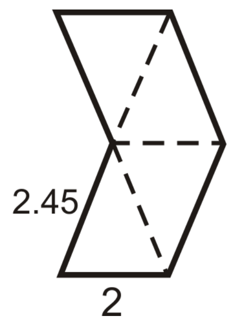
a. Make an open net (omit one base) of a cube, with 2 inch sides.



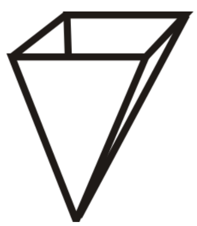
b. Cut out the net and tape up the sides to form an open cube.



c. Make an open net (no base) of a square pyramid, with lateral edges of 2.45 inches and base edges of 2 inches. This will make the overall height 2 inches.



d. Cut out the net and tape up the sides to form an open pyramid.

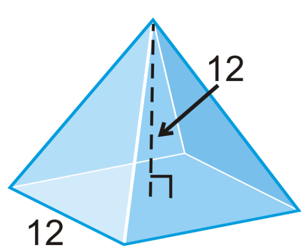


e. Fill the pyramid with dry rice. Then, dump the rice into the open cube. How many times do you have to repeat this to fill the cube?

Volume of a Pyramid: If B is the area of the base and h is the height, then the volume of a pyramid is $V = \frac{1}{3}Bh$.

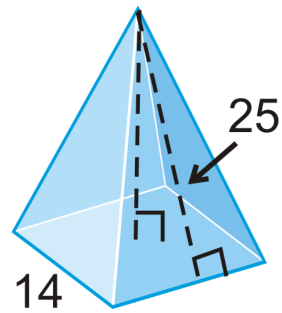
The investigation showed us that you would need to repeat this process three times to fill the cube. This means that the pyramid is one-third the volume of a prism with the same base.

Example 1: Find the volume of the pyramid.



Solution: $V = \frac{1}{3}(12^2)12 = 576\ units^3$

Example 2: Find the volume of the pyramid.

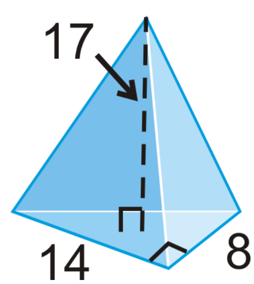


Solution: In this example, we are given the slant height. For volume, we need the height, so we need to use the Pythagorean Theorem to find it.

$$\begin{aligned}7^2 + h^2 &= 25^2\\ h^2 &= 576\\ h &= 24\end{aligned}$$

Using the height, the volume is $\frac{1}{3}(14^2)(24) = 1568\ units^3$.

Example 3: Find the volume of the pyramid.



Solution: The base of this pyramid is a right triangle. So, the area of the base is $\frac{1}{2}(14)(8) = 56\ units^2$.

$$V = \frac{1}{3}(56)(17) \approx 317.33\ units^3$$

Example 4: A rectangular pyramid has a base area of 56 cm^2 and a volume of 224 cm^3. What is the height of the pyramid?

Solution: The formula for the volume of a pyramid works for any pyramid, as long as you can find the area of the base.

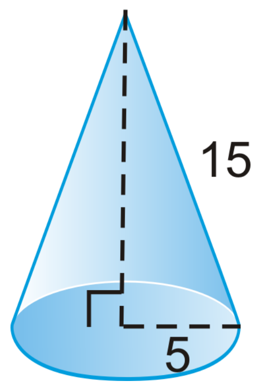
$$\begin{aligned}224 &= 56h\\ 4 &= h\end{aligned}$$

Volume of a Cone

The volume of cone has the same relationship with a cylinder as pyramid does with a prism. If the bases of a cone and a cylinder are the same, then the volume of a cone will be one-third the volume of the cylinder.

Volume of a Cone: If r is the radius of a cone and h is the height, then the volume is $V = \frac{1}{3}\pi r^2 h$.

Example 5: Find the volume of the cone.



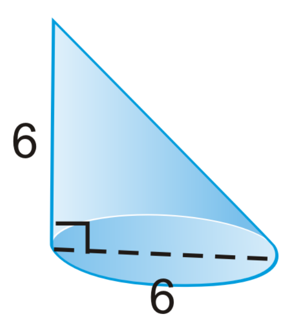
Solution: To find the volume, we need the height, so we have to use the Pythagorean Theorem.

$$\begin{aligned} 5^2 + h^2 &= 15^2 \\ h^2 &= 200 \\ h &= 10\sqrt{2} \end{aligned}$$

Now, we can find the volume.

$$V = \frac{1}{3}(5^2)\left(10\sqrt{2}\right)\pi \approx 370.24$$

Example 6: Find the volume of the cone.



Solution: Even though this doesn't look like the cone in Example 5, we can still find the volume in the same way. Use the *radius* in the formula.

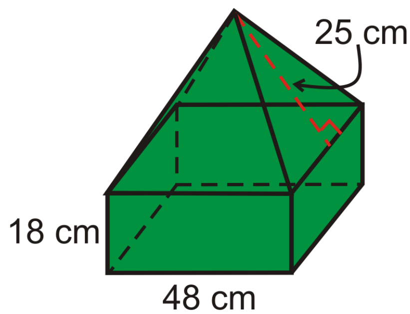
$$V = \frac{1}{3}\pi(3^2)(6) = 18\pi \approx 56.55$$

Example 7: The volume of a cone is $484\pi \ cm^3$ and the height is 12 cm. What is the radius?

Solution: Plug in what you know to the volume formula.

$$\begin{aligned}484\pi &= \frac{1}{3}\pi r^2(12)\\ 121 &= r^2\\ 11 &= r\end{aligned}$$

Example 8: Find the volume of the composite solid. All bases are squares.



Solution: This is a square prism with a square pyramid on top. Find the volume of each separeatly and then add them together to find the total volume. First, we need to find the height of the pyramid portion. The slant height is 25 and the edge is 48. Using have of the edge, we have a right triangle and we can use the Pythagorean Theorem. $h = \sqrt{25^2 - 24^2} = 7$

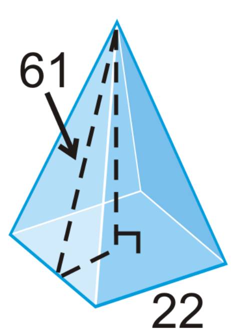
$$\begin{aligned}V_{prism} &= (48)(48)(18) = 41472 \ cm^3\\ V_{pyramid} &= \frac{1}{3}(48^2)(7) = 5376 \ cm^3\end{aligned}$$

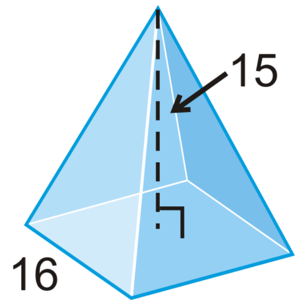
The total volume is $41472 + 5376 = 46,848 \ cm^3$.

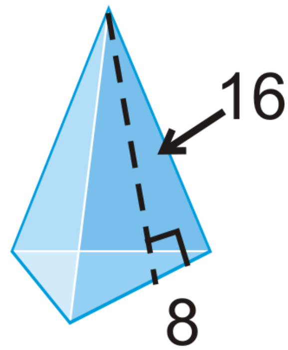
Know What? Revisited The original volume of the pyramid is $\frac{1}{3}(706^2)(407.5) \approx 67,704,223.33 \ ft^3$.

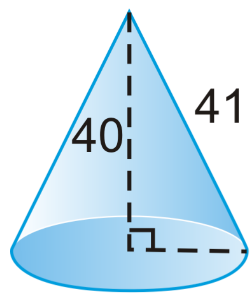
Review Questions

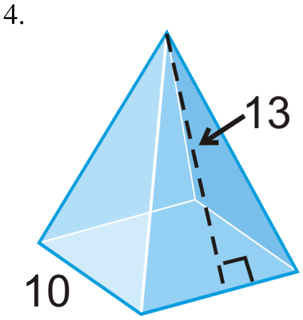
Find the volume of each regular pyramid and right cone. Round any decimal answers to the nearest hundredth. The bases of these pyramids are either squares or equilateral triangles.

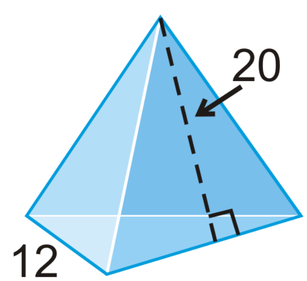
1. 

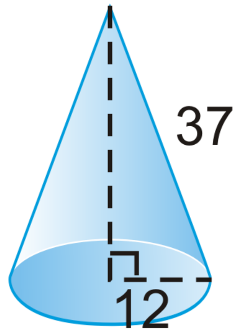
2. 

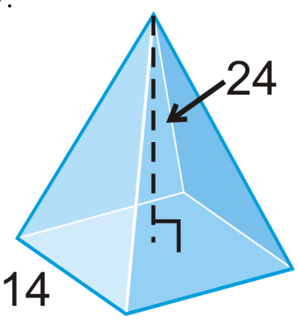
3. 

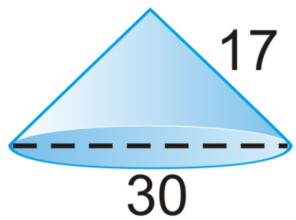
4. 

5. 

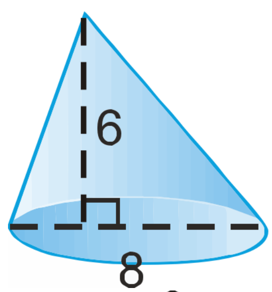
6. 

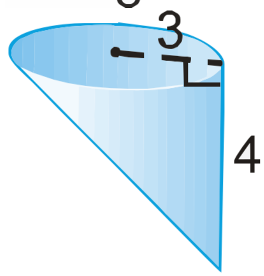
7. 

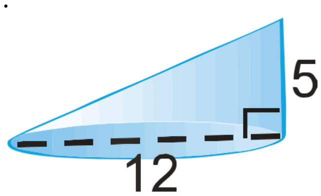
8. 

9. 

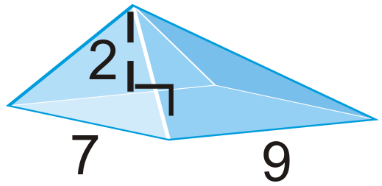
Find the volume of the following non-regular pyramids and cones. Round any decimal answers to the nearest hundredth.

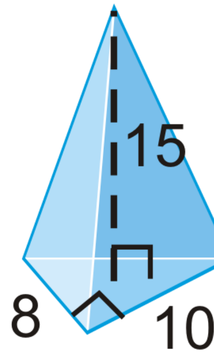
10. 

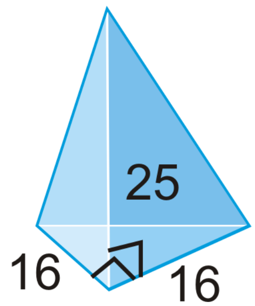
11. 

12. 

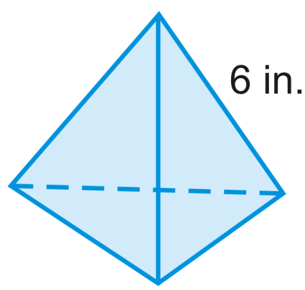
13. base is a rectangle



14. 

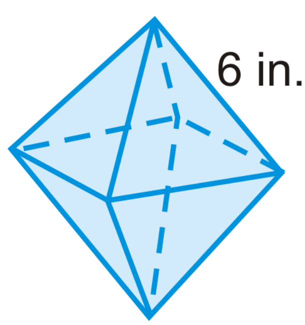
15. 

A ***regular tetrahedron*** has four equilateral triangles as its faces. Use the diagram to answer questions 16-19.



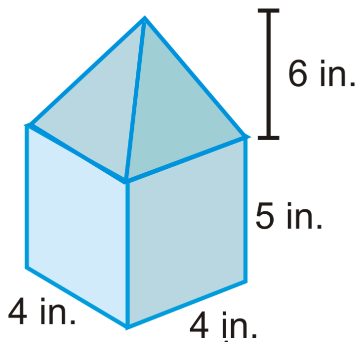
16. What is the area of the base of this regular etrahedron?
17. What is the height of this figure? Be careful!
18. Find the volume. Leave your answer in simplest radical form.
19. ***Challenge*** If the sides are length s, what is the volume?

A ***regular octahedron*** has eight equilateral triangles as its faces. Use the diagram to answer questions 19-21.

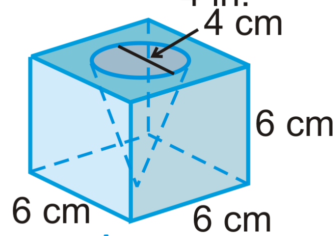


20. *Describe* how you would find the volume of this figure.
21. Find the volume. Leave your answer in simplest radical form.
22. ***Challenge*** If the sides are length s, what is the volume?
23. The volume of a square pyramid is 72 square inches and the base edge is 4 inches. What is the height?
24. If the volume of a cone is $30\pi\ cm^2$ and the radius is 5 cm, what is the height?
25. If the volume of a cone is $105\pi\ cm^2$ and the height is 35 cm, what is the radius?

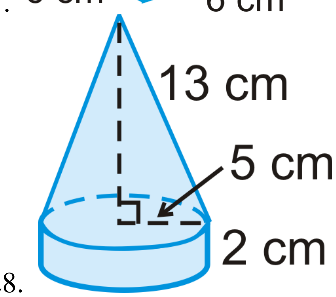
Find the volume of the composite solids. Round your answer to the nearest hundredth.



26.

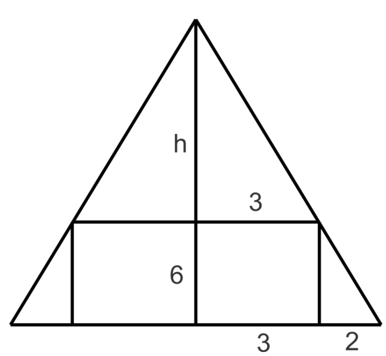


27.



28.

29. The ratio of the height to radius in a cone is 3:2. If the volume is $108\pi\ m^3$, find the height and radius of the cone.
30. A teepee is to be built such that there is a minimal cylindrical shaped central living space contained within the cone shape of diameter 6 ft and height 6 ft. If the radius of the entire teepee is 5 ft, find the total height of the teepee.



Review Queue Answers

a. $(8^2)(12) = 768\ in^3$
b. $(4^2)(12)\pi = 192\pi \approx 603.19$
c. The volume of the square prism is greater because the square base is larger than a circle with the same diameter as the square's edge.
d. Find slant height, $l = 13$. $SA = 100 + \frac{1}{2}(40)(13) = 360\ in^2$

11.6 Surface Area and Volume of Spheres

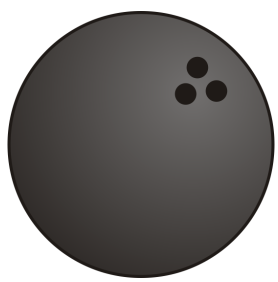
Learning Objectives

- Find the surface area of a sphere.
- Find the volume of a sphere.

Review Queue

a. List three spheres you would see in real life.
b. Find the area of a circle with a 6 cm radius.
c. Find the volume of a cylinder with the circle from #2 as the base and a height of 5 cm.
d. Find the volume of a cone with the circle from #2 as the base and a height of 5 cm.

Know What? A regulation bowling ball is a sphere that weighs between 12 and 16 pounds. The maximum circumference of a bowling ball is 27 inches. Using this number, find the radius of a bowling ball, its surface area and volume. You may assume the bowling ball does not have any finger holes. Round your answers to the nearest hundredth.

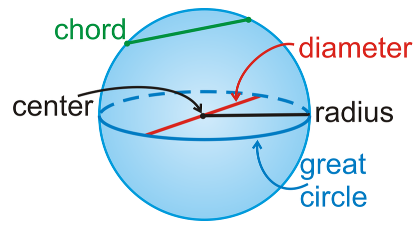


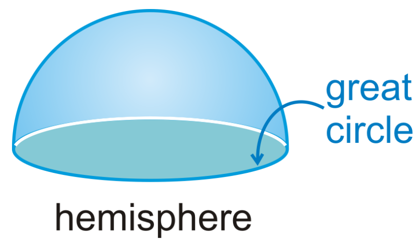
Defining a Sphere

A sphere is the last of the three-dimensional shapes that we will find the surface area and volume of. Think of a sphere as a three-dimensional circle. You have seen spheres in real-life countless times; tennis balls, basketballs, volleyballs, golf balls, and baseballs. Now we will analyze the parts of a sphere.

Sphere: The set of all points, in three-dimensional space, which are equidistant from a point.

A sphere has a ***center***, radius and diameter, just like a circle. The ***radius*** has an endpoint on the sphere and the other is on the center. The ***diameter*** must contain the center. If it does not, it is a ***chord***. The ***great circle*** is a plane that contains the diameter. It would be the largest circle cross section in a sphere. There are infinitely many great circles. ***The circumference of a sphere is the circumference of a great circle***. Every great circle divides a sphere into two congruent hemispheres, or two half spheres. Also like a circle, spheres can have tangent lines and secants. These are defined just like they are in a circle.





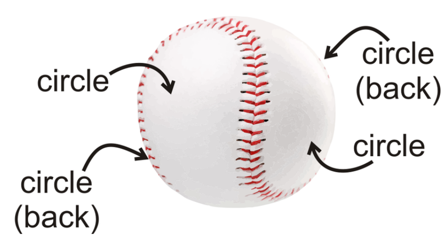
Example 1: The circumference of a sphere is 26π *feet*. What is the radius of the sphere?

Solution: The circumference is referring to the circumference of a great circle. Use $C = 2\pi r$.

$$\begin{aligned} 2\pi r &= 26\pi \\ r &= 13\ ft. \end{aligned}$$

Surface Area of a Sphere

One way to find the formula for the surface area of a sphere is to look at a baseball. We can best *approximate* the cover of the baseball by 4 circles. The area of a circle is πr^2, so the surface area of a sphere is $4\pi r^2$. While the covers of a baseball are not four perfect circles, they are stretched and skewed.



Another way to show the surface area of a sphere is to watch the link by Russell Knightley, http://www.rkm.com.au/ANIMATIONS/animation-Sphere-Surface-Area-Derivation.html . It is a great visual interpretation of the formula.

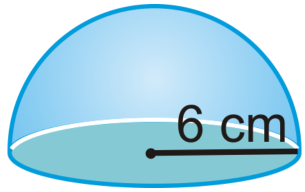
Surface Area of a Sphere: If r is the radius, then the surface area of a sphere is $SA = 4\pi r^2$.

Example 2: Find the surface area of a sphere with a radius of 14 feet.

Solution: Use the formula, $r = 14\ ft$.

$$\begin{aligned} SA &= 4\pi(14)^2 \\ &= 784\pi\ ft^2 \end{aligned}$$

Example 3: Find the surface area of the figure below.



Solution: This is a hemisphere. Be careful when finding the surface area of a hemisphere because you need to include the area of the base. If the question asked for the *lateral surface area*, then your answer would *not* include the bottom.

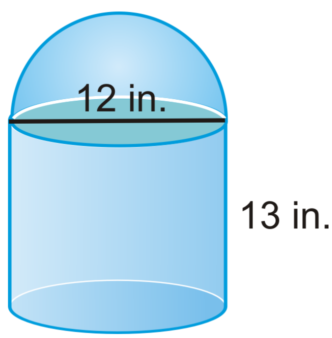
$$\begin{aligned} SA &= \pi r^2 + \frac{1}{2}4\pi r^2 \\ &= \pi(6^2) + 2\pi(6^2) \\ &= 36\pi + 72\pi = 108\pi\ cm^2 \end{aligned}$$

Example 4: The surface area of a sphere is $100\pi\ in^2$. What is the radius?

Solution: Plug in what you know to the formula and solve for r.

$$\begin{aligned} 100\pi &= 4\pi r^2 \\ 25 &= r^2 \\ 5 &= r \end{aligned}$$

Example 5: Find the surface area of the following solid.

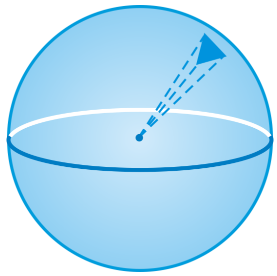


Solution: This solid is a cylinder with a hemisphere on top. Because it is one fluid solid, we would not include the bottom of the hemisphere or the top of the cylinder because they are no longer on the surface of the solid. Below, "*LA*" stands for *lateral area*.

$$\begin{aligned} SA &= LA_{cylinder} + LA_{hemisphere} + A_{base\ circle} \\ &= \pi rh + \frac{1}{2}4\pi r^2 + \pi r^2 \\ &= \pi(6)(13) + 2\pi 6^2 + \pi 6^2 \\ &= 78\pi + 72\pi + 36\pi \\ &= 186\pi\ in^2 \end{aligned}$$

Volume of a Sphere

A sphere can be thought of as a regular polyhedron with an infinite number of congruent regular polygon faces. As n, the number of faces increases to an infinite number, the figure approaches becoming a sphere. So a sphere can be thought of as a polyhedron with an infinite number faces. Each of those faces is the base of a pyramid whose vertex is located at the center of the sphere. Each of the pyramids that make up the sphere would be congruent to the pyramid shown. The volume of this pyramid is given by $V = \frac{1}{3}Bh$.



To find the volume of the sphere, you need to add up the volumes of an infinite number of infinitely small pyramids.

$$\begin{aligned} V(all\ pyramids) &= V_1 + V_2 + V_3 + \ldots + V_n \\ &= \frac{1}{3}(B_1h + B_2h + B_3h + \ldots + B_nh) \\ &= \frac{1}{3}h(B_1 + B_2 + B_3 + \ldots + B_n) \end{aligned}$$

The sum of all of the bases of the pyramids is the surface area of the sphere. Since you know that the surface area of the sphere is $4\pi r^2$, you can substitute this quantity into the equation above.

$$= \frac{1}{3}h\left(4\pi r^2\right)$$

In the picture above, we can see that the height of each pyramid is the radius, so $h = r$.

$$\begin{aligned} &= \frac{4}{3}r(\pi r^2) \\ &= \frac{4}{3}\pi r^3 \end{aligned}$$

To see an animation of the volume of a sphere, see http://www.rkm.com.au/ANIMATIONS/animation-Sphere-Volume-Derivation.html by Russell Knightley. It is a slightly different interpretation than our derivation.

Volume of a Sphere: If a sphere has a radius r, then the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Example 6: Find the volume of a sphere with a radius of 9 m.

Solution: Use the formula above.

$$
\begin{aligned}
V &= \frac{4}{3}\pi 6^3 \\
&= \frac{4}{3}\pi(216) \\
&= 288\pi
\end{aligned}
$$

Example 7: A sphere has a volume of 14137.167 ft^3, what is the radius?

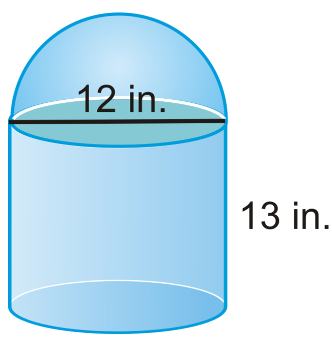
Solution: Because we have a decimal, our radius might be an approximation. Plug in what you know to the formula and solve for r.

$$
\begin{aligned}
14137.167 &= \frac{4}{3}\pi r^3 \\
\frac{3}{4\pi} \cdot 14137.167 &= r^3 \\
3375 &= r^3
\end{aligned}
$$

At this point, you will need to take the ***cubed root*** of 3375. Depending on your calculator, you can use the $\sqrt[3]{x}$ function or $\wedge\left(\frac{1}{3}\right)$. The cubed root is the inverse of cubing a number, just like the square root is the inverse, or how you undo, the square of a number.

$$\sqrt[3]{3375} = 15 = r \qquad \text{The radius is 15 } ft.$$

Example 8: Find the volume of the following solid.



Solution: To find the volume of this solid, we need the volume of a cylinder and the volume of the hemisphere.

$$
\begin{aligned}
V_{cylinder} &= \pi 6^2(13) = 78\pi \\
V_{hemisphere} &= \frac{1}{2}\left(\frac{4}{3}\pi 6^3\right) = 36\pi \\
V_{total} &= 78\pi + 36\pi = 114\pi \ in^3
\end{aligned}
$$

Know What? Revisited If the maximum circumference of a bowling ball is 27 inches, then the maximum radius would be $27 = 2\pi r$, or $r = 4.30$ inches. Therefore, the surface area would be $4\pi 4.3^2 \approx 232.35\ in^2$, and the volume would be $\frac{4}{3}\pi\ 4.3^3 \approx 333.04\ in^3$. The weight of the bowling ball refers to its density, how heavy something is. The volume of the ball tells us how much it can hold.

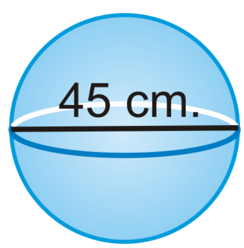
Review Questions

1. Are there any cross-sections of a sphere that are not a circle? Explain your answer.

Find the surface area and volume of a sphere with: (Leave your answer in terms of π)

2. a radius of 8 in.
3. a diameter of 18 cm.
4. a radius of 20 ft.
5. a diameter of 4 m.
6. a radius of 15 ft.
7. a diameter of 32 in.
8. a circumference of $26\pi\ cm$.
9. a circumference of $50\pi\ yds$.
10. The surface area of a sphere is $121\pi\ in^2$. What is the radius?
11. The volume of a sphere is $47916\pi\ m^3$. What is the radius?
12. The surface area of a sphere is $4\pi\ ft^2$. What is the volume?
13. The volume of a sphere is $36\pi\ mi^3$. What is the surface area?
14. Find the radius of the sphere that has a volume of $335\ cm^3$. Round your answer to the nearest hundredth.
15. Find the radius of the sphere that has a surface area $225\pi\ ft^2$.

Find the surface area of the following shapes. Leave your answers in terms of π.



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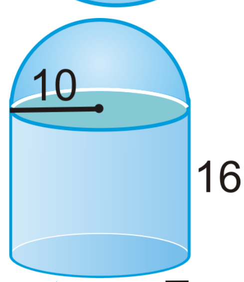

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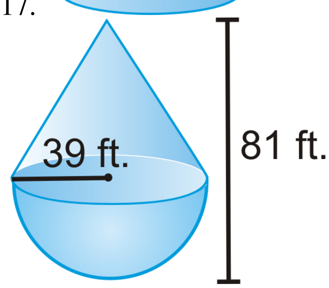
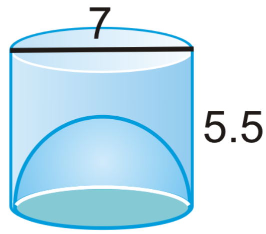


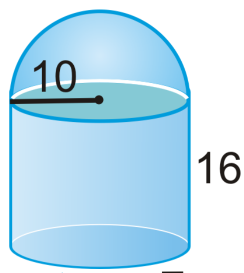
18.

19. You may assume the bottom is *open*.



Find the volume of the following shapes. Round your answers to the nearest hundredth.

20.



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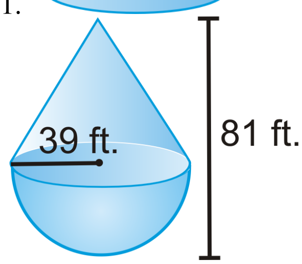

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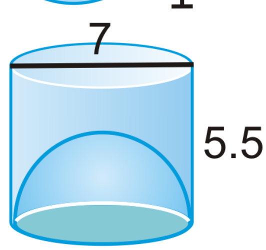
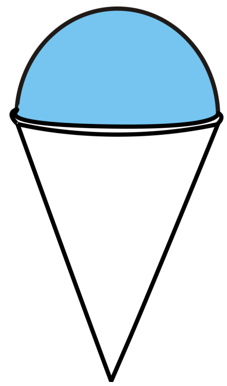


23.

24. A sphere has a radius of 5 cm. A right cylinder has the same radius and volume. Find the height and total surface area of the cylinder.
25. Tennis balls with a 3 inch diameter are sold in cans of three. The can is a cylinder. Assume the balls touch the can on the sides, top and bottom. What is the volume of the space *not* occupied by the tennis balls? Round your answer to the nearest hundredth.

26. One hot day at a fair you buy yourself a snow cone. The height of the cone shaped container is 5 in and its radius is 2 in. The shaved ice is perfectly rounded on top forming a hemisphere. What is the volume of the ice in your frozen treat? If the ice melts at a rate of 2 cm^3 per minute, how long do you have to eat your treat before it all melts? Give your answer to the nearest minute.



Multi-Step Problems

27. a. What is the surface area of a cylinder?
    b. Adjust your answer from part a for the case where $r = h$.
    c. What is the surface area of a sphere?
    d. What is the relationship between your answers to parts b and c? Can you explain this?
28. At the age of 81, Mr. Luke Roberts began collecting string. He had a ball of string 3 feet in diameter.
    a. Find the volume of Mr. Roberts' ball of string in cubic inches.
    b. Assuming that each cubic inch weighs 0.03 pounds, find the weight of his ball of string.
    c. To the nearest inch, how big (diameter) would a 1 ton ball of string be? $(1\ ton = 2000\ lbs)$

For problems 29-31, use the fact that the earth's radius is approximately 4,000 miles.

29. Find the length of the equator.
30. Find the surface area of earth, rounding your answer to the nearest million square miles.
31. Find the volume of the earth, rounding your answer to the nearest billion cubic miles.

Review Queue Answers

a. *Answers will vary.* Possibilities are any type of ball, certain lights, or the 76/Unical orb.
b. 36π
c. 180π
d. 60π

11.7 Exploring Similar Solids

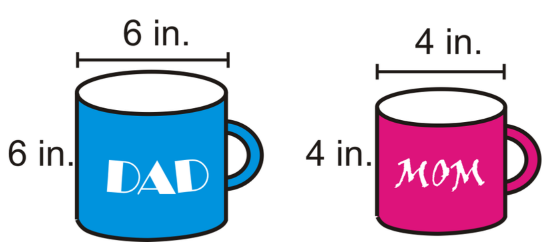
Learning Objectives

- Find the relationship between similar solids and their surface areas and volumes.

Review Queue

a. We know that every circle is similar, is every sphere similar?
b. Find the volume of a sphere with a 12 in radius. Leave your answer in terms of π.
c. Find the volume of a sphere with a 3 in radius. Leave your answer in terms of π.
d. Find the scale factor of the spheres from #2 and #3. Then find the ratio of the volumes and reduce it. What do you notice?
e. Two squares have a scale factor of 2:3. What is the ratio of their areas?
f. The smaller square from #5 has an area of 16 cm^2. What is the area of the larger square?
g. The ratio of the areas of two similar triangles is 1:25. The height of the larger triangle is 20 cm, what is the height of the smaller triangle?

Know What? Your mom and dad have cylindrical coffee mugs with the dimensions to the right. Are the mugs similar? (You may ignore the handles.) If the mugs are similar, find the volume of each, the scale factor and the ratio of the volumes.

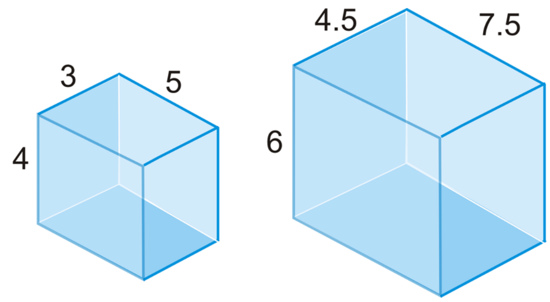


Similar Solids

Recall that two shapes are similar if all the corresponding angles are congruent and the corresponding sides are proportional.

Similar Solids: Two solids are similar if and only if they are the same type of solid and their corresponding linear measures (radii, heights, base lengths, etc.) are proportional.

Example 1: Are the two rectangular prisms similar? How do you know?

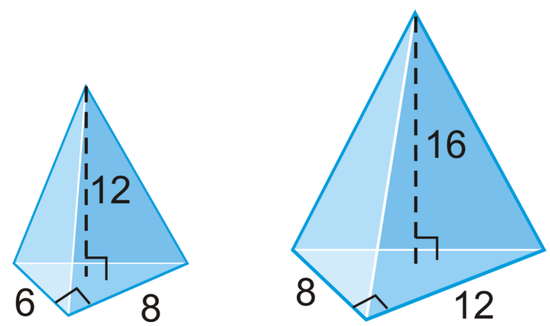


Solution: Match up the corresponding heights, widths, and lengths to see if the rectangular prisms are proportional.

$$\frac{small\ prism}{large\ prism} = \frac{3}{4.5} = \frac{4}{6} = \frac{5}{7.5}$$

The congruent ratios tell us the two prisms are similar.

Example 2: Determine if the two triangular pyramids similar.



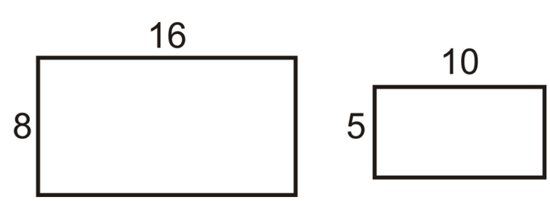
Solution: Just like Example 1, let's match up the corresponding parts.

$\frac{6}{8} = \frac{12}{16} = \frac{3}{4}$ however, $\frac{8}{12} = \frac{2}{3}$.

Because one of the base lengths is not in the same proportion as the other two lengths, these right triangle pyramids are not similar.

Surface Areas of Similar Solids

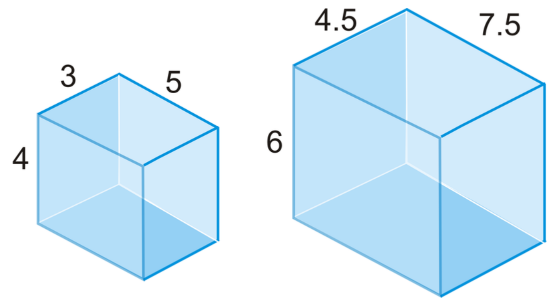
Recall that ***when two shapes are similar, the ratio of the area is a square of the scale factor.***



For example, the two rectangles to the left are similar because their sides are in a ratio of 5:8. The area of the larger rectangle is $8(16) = 128\ units^2$ and the area of the smaller rectangle is $5(10) = 50\ units^2$. If we compare the areas in a ratio, it is $50 : 128 = 25 : 64 = 5^2 = 8^2$.

So, what happens with the surface areas of two similar solids? Let's look at Example 1 again.

Example 3: Find the surface area of the two similar rectangular prisms.



Solution:

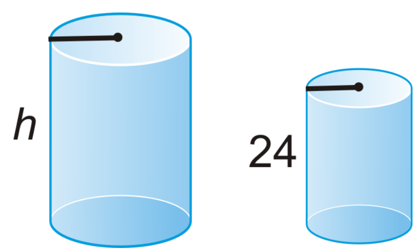
$$\begin{aligned} SA_{smaller} &= 2(4 \cdot 3) + 2(4 \cdot 5) + 2(3 \cdot 5) \\ &= 24 + 40 + 30 = 94 \ units^2 \end{aligned}$$

$$\begin{aligned} SA_{larger} &= 2(6 \cdot 4.5) + 2(4.5 \cdot 7.5) + 2(6 \cdot 7.5) \\ &= 54 + 67.5 + 90 = 211.5 \ units^2 \end{aligned}$$

Now, find the ratio of the areas. $\frac{94}{211.5} = \frac{4}{9} = \frac{2^2}{3^2}$. The sides are in a ratio of $\frac{4}{6} = \frac{2}{3}$, so the surface areas have the same relationship as the areas of two similar shapes.

Surface Area Ratio: If two solids are similar with a scale factor of $\frac{a}{b}$, then the surface areas are in a ratio of $\left(\frac{a}{b}\right)^2$.

Example 4: Two similar cylinders are below. If the ratio of the areas is 16:25, what is the height of the taller cylinder?



Solution: First, we need to take the square root of the area ratio to find the scale factor, $\sqrt{\frac{16}{25}} = \frac{4}{5}$. Now we can set up a proportion to find h.

$$\begin{aligned} \frac{4}{5} &= \frac{24}{h} \\ 4h &= 120 \\ h &= 30 \end{aligned}$$

Example 5: Using the cylinders from Example 4, if the surface area of the smaller cylinder is $1536\pi \ cm^2$, what is the surface area of the larger cylinder?

Solution: Set up a proportion using the ratio of the areas, 16:25.

$$\frac{16}{25} = \frac{1536\pi}{A}$$
$$16A = 38400\pi$$
$$A = 2400\pi\ cm^2$$

Volumes of Similar Solids

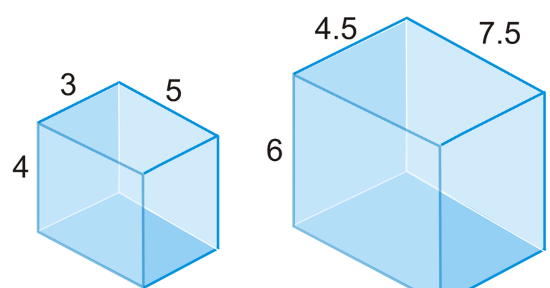
Let's look at what we know about similar solids so far.

TABLE 11.3:

	Ratios	*Units*
Scale Factor	$\frac{a}{b}$	in, ft, cm, m, etc.
Ratio of the Surface Areas	$\left(\frac{a}{b}\right)^2$	in^2, ft^2, cm^2, m^2, etc.
Ratio of the Volumes	??	in^3, ft^3, cm^3, m^3, etc.

It looks as though there is a pattern. If the ratio of the volumes follows the pattern from above, it should be the cube of the scale factor. We will do an example and test our theory.

Example 6: Find the volume of the following rectangular prisms. Then, find the ratio of the volumes.



Solution:

$$V_{smaller} = 3(4)(5) = 60$$
$$V_{larger} = 4.5(6)(7.5) = 202.5$$

The ratio is $\frac{60}{202.5}$, which reduces to $\frac{8}{27} = \frac{2^3}{3^3}$.

It seems as though our prediction based on the patterns is correct.

Volume Ratio: If two solids are similar with a scale factor of $\frac{a}{b}$, then the volumes are in a ratio of $\left(\frac{a}{b}\right)^3$.

Example 7: Two spheres have radii in a ratio of 3:4. What is the ratio of their volumes?

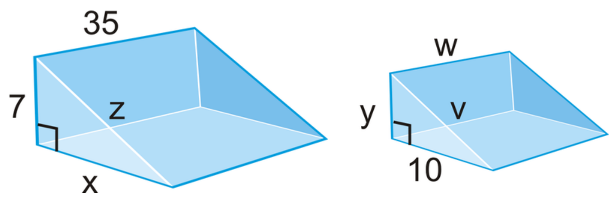
Solution: If we cube 3 and 4, we will have the ratio of the volumes. Therefore, $3^3 : 4^3$ or 27:64 is the ratio of the volumes.

Example 8: If the ratio of the volumes of two similar prisms is 125:8, what is their scale factor?

Solution: This example is the opposite of the previous example. We need to take the cubed root of 125 and 8 to find the scale factor.

$$\sqrt[3]{125} : \sqrt[3]{8} = 5 : 2$$

Example 9: Two similar right triangle prisms are below. If the ratio of the volumes is 343:125, find the missing sides in both figures.



Solution: If the ratio of the volumes is 343:125, then the scale factor is 7:5, the cubed root of each. With the scale factor, we can now set up several proportions.

$$\frac{7}{5} = \frac{7}{y} \qquad \frac{7}{5} = \frac{x}{10} \qquad \frac{7}{5} = \frac{35}{w} \qquad 7^2 + x^2 = z^2 \qquad \frac{7}{5} = \frac{z}{v}$$

$$y = 5 \qquad x = 14 \qquad w = 25 \qquad 7^2 + 14^2 = z^2$$

$$z = \sqrt{245} = 7\sqrt{5} \qquad \frac{7}{5} = \frac{7\sqrt{5}}{v} \rightarrow v = 5\sqrt{5}$$

Example 10: The ratio of the surface areas of two similar cylinders is 16:81. If the volume of the smaller cylinder is $96\pi\ in^3$, what is the volume of the larger cylinder?

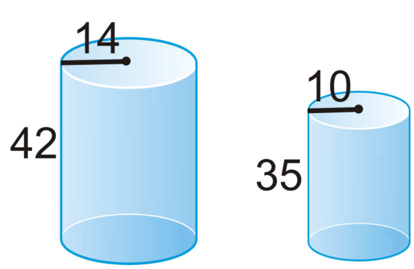
Solution: First we need to find the scale factor from the ratio of the surface areas. If we take the square root of both numbers, we have that the ratio is 4:9. Now, we need cube this to find the ratio of the volumes, $4^3 : 9^3 = 64 : 729$. At this point we can set up a proportion to solve for the volume of the larger cylinder.

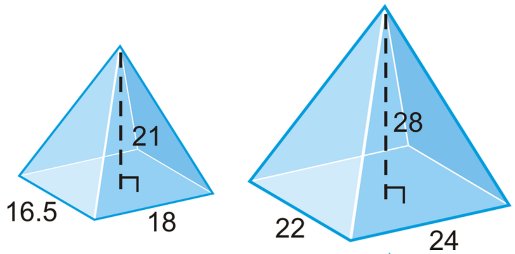
$$\frac{64}{729} = \frac{96\pi}{V}$$
$$64V = 69984\pi$$
$$V = 1093.5\pi\ in^3$$

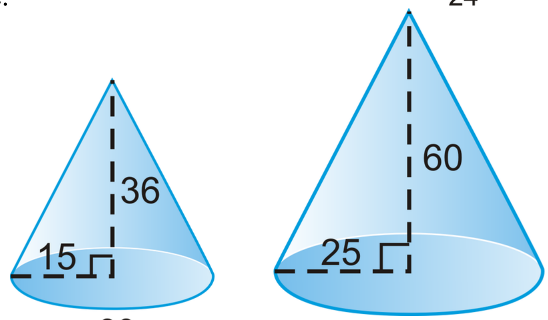
Know What? Revisited The coffee mugs are similar because the heights and radii are in a ratio of 2:3, which is also their scale factor. The volume of Dad's mug is $54\pi\ in^3$ and Mom's mug is $16\pi\ in^3$. The ratio of the volumes is $54\pi : 16\pi$, which reduces to 8:27.

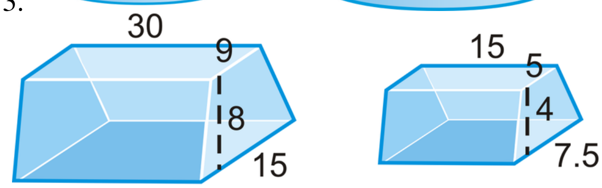
Review Questions

Determine if each pair of right solids are similar. *Explain* your reasoning.

1. 

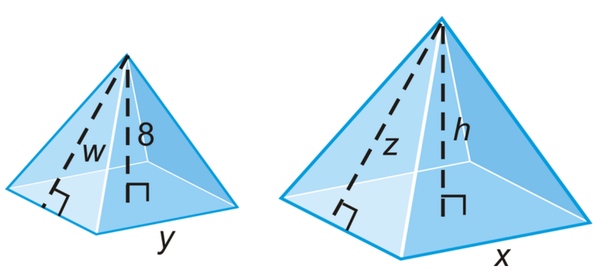
2. 

3. 

4. 

5. Are all cubes similar? Why or why not?
6. Two prisms have a scale factor of 1:4. What is the ratio of their surface areas?
7. Two pyramids have a scale factor of 2:7. What is the ratio of their volumes?
8. Two spheres have radii of 5 and 9. What is the ratio of their volumes?
9. The surface area of two similar cones is in a ratio of 64:121. What is the scale factor?
10. The volume of two hemispheres is in a ratio of 125:1728. What is the scale factor?
11. A cone has a volume of 15π and is similar to another larger cone. If the scale factor is 5:9, what is the volume of the larger cone?
12. A cube has sides of length x and is enlarged so that the sides are $4x$. How does the volume change?
13. The ratio of the volumes of two similar pyramids is 8:27. What is the ratio of their total surface areas?
14. The ratio of the volumes of two tetrahedrons is 1000:1. The smaller tetrahedron has a side of length 6 cm. What is the side length of the larger tetrahedron?
15. The ratio of the surface areas of two cubes is 64:225. If the volume of the smaller cube is 13824 m^3, what is the volume of the larger cube?

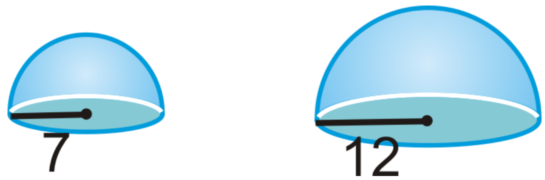
Below are two similar square pyramids with a volume ratio of 8:27. The base lengths are equal to the heights. Use this to answer questions 16-21.



16. What is the scale factor?
17. What is the ratio of the surface areas?
18. Find h, x and y.
19. Find w and z.

20. Find the volume of both pyramids.
21. Find the *lateral* surface area of both pyramids.

Use the hemispheres below to answer questions 22-25.



22. Are the two hemispheres similar? How do you know?
23. Find the ratio of the surface areas and volumes.
24. Find the *lateral* surface areas of both hemispheres.
25. Determine the ratio of the lateral surface areas for the hemispheres. Is it the same as the ratio of the total surface area? Why or why not?

Animal A and animal B are similar (meaning the size and shape of their bones and bodies are similar) and *the strength of their respective bones are proportional to the cross sectional area of their bones*. Answer the following questions given that the ratio of the height of animal A to the height of animal B is 3:5. You may assume the lengths of their bones are in the same ratio.

26. Find the ratio of the strengths of the bones. How much stronger are the bones in animal B?
27. If their weights are proportional to their volumes, find the ratio of their weights.
28. Which animal has a skeleton more capable of supporting its own weight? Explain.

Two sizes of cans of beans are similar. The thickness of the walls and bases are the same in both cans. The ratio of their surface areas is 4:9.

29. If the surface area of the smaller can is 36 sq in, what is the surface area of the larger can?
30. If the sheet metal used to make the cans costs \$0.006 per square inch, how much does it cost to make each can?
31. What is the ratio of their volumes?
32. If the smaller can is sold for \$0.85 and the larger can is sold for \$2.50, which is a better deal?

Review Queue Answers

a. Yes, every sphere is similar because the similarity only depends on one length, the radius.
b. $\frac{4}{3}12^3\pi = 2304\pi\ in^3$
c. $\frac{4}{3}3^3\pi = 27\pi\ in^3$
d. The scale factor is 4:1, the volume ratio is 2304:36 or 64:1
e. $\frac{4}{9}$
f. $\frac{4}{9} = \frac{16}{A} \rightarrow A = 36\ cm^2$
g. $\frac{1}{5} = \frac{x}{20} \rightarrow x = 4\ cm$

11.8 Chapter 11 Review

Keywords, Theorems, Formulas

Polyhedron
A 3-dimensional figure that is formed by polygons that enclose a region in space.

Face
Each polygon in a polyhedron is called a ***face***.

Edge
The line segment where two faces intersect is called an ***edge***

Vertex
the point of intersection of two edges is a ***vertex***.

Prism
A polyhedron with two congruent bases, in parallel planes, and the lateral sides are rectangles.

Pyramid
A polyhedron with one base and all the lateral sides meet at a common vertex. The lateral sides are triangles.

Euler's Theorem
The number of faces (F), vertices (V), and edges (E) of a polyhedron can be related such that $F+V=E+2$.

Regular Polyhedron
A polyhedron where all the faces are congruent regular polygons.

Regular Tetrahedron
A 4-faced polyhedron where all the faces are equilateral triangles.

Cube
A 6-faced polyhedron where all the faces are squares.

Regular Octahedron
An 8-faced polyhedron where all the faces are equilateral triangles.

Regular Dodecahedron
A 12-faced polyhedron where all the faces are regular pentagons.

Regular Icosahedron
A 20-faced polyhedron where all the faces are equilateral triangles.

Cross-Section
The intersection of a plane with a solid.

Net
An unfolded, flat representation of the sides of a three-dimensional shape.

Lateral Face
A face that is not the base.

Lateral Edge
The edges between the ***lateral faces*** are called ***lateral edges***.

Base Edge
The edges between the base and the lateral faces are called ***base edges***.

Right Prism
All prisms are named by their bases, so the prism to the right is a pentagonal prism. This particular prism is called a ***right prism***

Oblique Prism
Oblique prisms lean to one side or the other and the height is outside the prism.

Surface Area
The sum of the areas of the faces.

Lateral Area
The sum of the areas of the ***lateral*** faces.

Surface Area of a Right Prism
The surface area of a right prism is the sum of the area of the bases and the area of each rectangular lateral face.

Cylinder
A solid with congruent circular bases that are in parallel planes. The space between the circles is enclosed.

Surface Area of a Right Cylinder
If r is the radius of the base and h is the height of the cylinder, then the surface area is $SA = 2\pi r^2 + 2\pi rh$.

Surface Area of a Regular Pyramid
If B is the area of the base and P is the perimeter of the base and l is the slant height, then $SA = B + \frac{1}{2}Pl$.

Cone
A solid with a circular base and sides taper up towards a common vertex.

Slant Height
All regular pyramids also have a ***slant height*** that is the height of a lateral face. Because of the nature of regular pyramids, ***all slant heights are congruent***. A non-regular pyramid does not have a slant height.

Surface Area of a Right Cone
The surface area of a right cone with slant height l and base radius r is $SA = \pi r^2 + \pi rl$.

Volume
The measure of how much space a three-dimensional figure occupies.

Volume of a Cube Postulate
The volume of a cube is the cube of the length of its side, or s^3.

Volume Congruence Postulate
If two solids are congruent, then their volumes are congruent.

Volume Addition Postulate
The volume of a solid is the sum of the volumes of all of its non-overlapping parts.

Volume of a Rectangular Prism
If a rectangular prism is h units high, w units wide, and l units long, then its volume is $V = l \cdot w \cdot h$.

Volume of a Prism
If the area of the base of a prism is B and the height is h, then the volume is $V = B \cdot h$.

Cavalieri's Principle
If two solids have the same height and the same cross-sectional area at every level, then they will have the same volume.

Volume of a Cylinder
If the height of a cylinder is h and the radius is r, then the volume would be $V = \pi r^2 h$.

Volume of a Pyramid
If B is the area of the base and h is the height, then the volume of a pyramid is $V = \frac{1}{3}Bh$.

Volume of a Cone
If r is the radius of a cone and h is the height, then the volume is $V = \frac{1}{3}\pi r^2 h$.

Sphere
The set of all points, in three-dimensional space, which are equidistant from a point.

Great Circle
The ***great circle*** is a plane that contains the diameter.

Surface Area of a Sphere
If r is the radius, then the surface area of a sphere is $SA = 4\pi r^2$.

Volume of a Sphere
If a sphere has a radius r, then the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

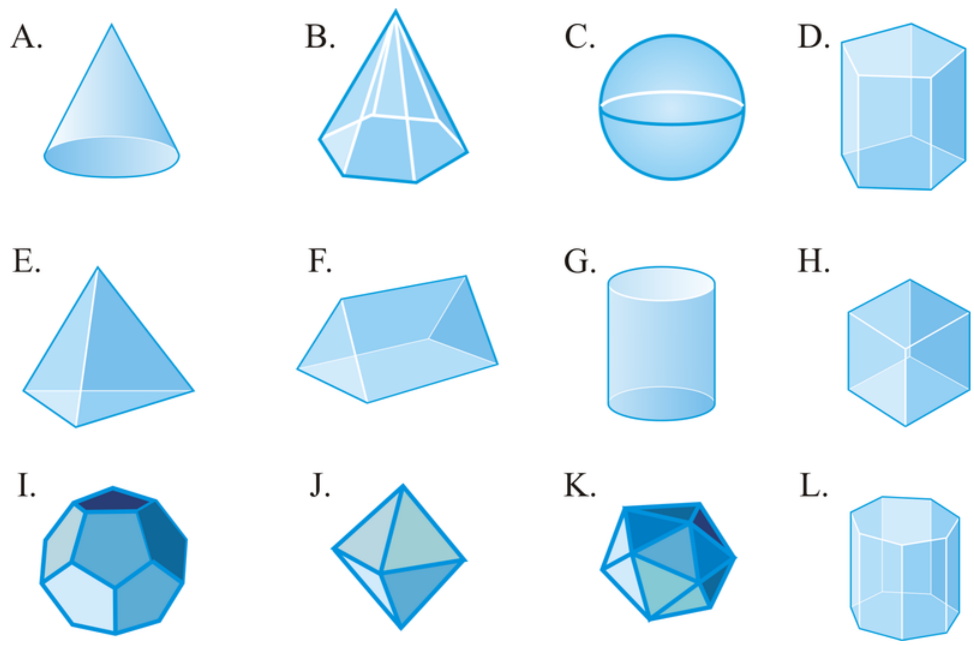
Similar Solids
Two solids are similar if and only if they are the same type of solid and their corresponding linear measures (radii, heights, base lengths, etc.) are proportional.

Surface Area Ratio
If two solids are similar with a scale factor of $\frac{a}{b}$, then the surface areas are in a ratio of $\left(\frac{a}{b}\right)^2$.

Volume Ratio
If two solids are similar with a scale factor of $\frac{a}{b}$, then the volumes are in a ratio of $\left(\frac{a}{b}\right)^3$.

Review Questions

Match the shape with the correct name.



1. Triangular Prism
2. Icosahedron
3. Cylinder
4. Cone
5. Tetrahedron
6. Pentagonal Prism
7. Octahedron
8. Hexagonal Pyramid
9. Octagonal Prism
10. Sphere
11. Cube
12. Dodecahedron

Match the formula with its description.

13. Volume of a Prism - A. $\frac{1}{3}\pi r^2 h$
14. Volume of a Pyramid - B. $\pi r^2 h$
15. Volume of a Cone - C. $4\pi r^2$
16. Volume of a Cylinder - D. $\frac{4}{3}\pi r^3$
17. Volume of a Sphere - E. $\pi r^2 + \pi rl$
18. Surface Area of a Prism - F. $2\pi r^2 + 2\pi rh$
19. Surface Area of a Pyramid - G. $\frac{1}{3}Bh$
20. Surface Area of a Cone - H. Bh
21. Surface Area of a Cylinder - I. $B + \frac{1}{2}Pl$
22. Surface Area of a Sphere - J. The sum of the area of the bases and the area of each rectangular lateral face.

Texas Instruments Resources

In the CK-12 Texas Instruments Geometry FlexBook, there are graphing calculator activities designed to supplement the objectives for some of the lessons in this chapter. See http://www.ck12.org/flexr/chapter/9696 .